BLACK Worksheet 1

EQUIVALENT FRACTIONS
Find 3 equivalent fractions for each of the following:

1.	$\frac{1}{4}$	$\frac{2}{8}$	$\frac{3}{12}$	$\frac{4}{16}$	6.	$\frac{3}{7}$	$\frac{6}{14}$	$\frac{9}{21}$	$\frac{12}{28}$
2.	$\frac{1}{3}$	$\frac{2}{6}$	$\frac{3}{9}$	$\frac{4}{12}$	7.	$\frac{4}{9}$	$\frac{8}{18}$	$\frac{12}{27}$	$\frac{16}{36}$
3.	$\frac{1}{5}$	$\frac{2}{10}$	$\frac{3}{15}$	$\frac{4}{20}$	8.	$\frac{5}{8}$	$\frac{10}{16}$	$\frac{15}{24}$	$\frac{20}{32}$
4.	$\frac{2}{3}$	$\frac{4}{6}$	$\frac{6}{9}$	$\frac{8}{12}$	9.	$\frac{5}{7}$	$\frac{6}{14}$	$\frac{9}{21}$	$\frac{12}{28}$
5.	$\frac{3}{4}$	$\frac{6}{8}$	$\frac{9}{12}$	$\frac{12}{16}$	10.	$\frac{5}{6}$	$\frac{10}{12}$	$\frac{15}{18}$	$\frac{20}{24}$

SMALLEST TO BIGGEST
Group these numbers from smallest to largest:

PUZZLE

1.	1.01, 1.1, 1.101	*1.01, 1.1, 1.101*
2.	2.43, 2.34, 2.034	*2.034, 2.34, 2.43*
3.	34.16, 34.016, 34.61	*34.016, 34.16, 34.61*
4.	21.27, 21.72, 21.072	*21.072, 21.27, 21.72*
5.	13.75, 1.375, 137.6	*1.375, 13.75, 137.5*
6.	141.2, 14.12, 14.21	*14.12, 14.21, 141.2*
7.	20.71, 207.1, 201.7	*20.71, 201.7, 207.1*
8.	0.456, 0.465, 0.645	*0.456, 0.465, 0.645*
9.	0.017, 0.0099, 0.071	
10.	1.008, 1.010, 1.001	*1.001, 1.008, 1.010*
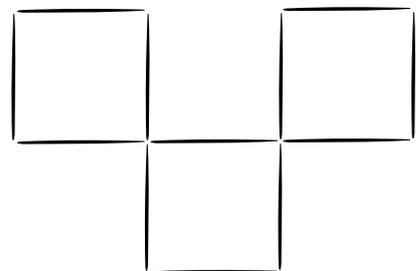
The arrangement above is made of 17 sticks.

Remove 5 sticks to leave 3 squares the same size as the original.

NUMERACY STRATEGIES (ADDITION)

Add by breaking down into powers of 10.

e.g. 38 + 73   = 30 + 8 + 70 + 3

                 = 100 + 11

                 = 111

1.   51 + 19 = *70*

2.   23 + 85 = *108*

3.   121 + 73 = *194*

4.   173 + 24 = *197*

5.   135 + 48 = *183*

6.   214 + 135 = *349*

7.   156 + 281 = *437*

8.   1321 + 1525 = *2846*

9.   2715 + 1272 = *3987*

10.   1731 + 2256 = *3987*

5 QUICK QUESTIONS

Use any strategy but not a calculator.

1.   143 + 215 = *358*

2.   67 - 34 = *33*

3.   31 × 4 = *124*

4.   48 ÷ 6 = *8*

5.   Round 3.926 to 1 decimal place.

        = *3.9*

NUMERACY STRATEGIES (MULTIPLICATION)

Multiply by breaking down into easier parts. e.g. $7 \times 14 = 7 \times (10 + 4)$
$$= 7 \times 10 + 7 \times 4$$
$$= 70 \quad + 28$$
$$= 98$$

1. $8 \times 13 = 104$

2. $5 \times 14 = 70$

3. $7 \times 19 = 133$

4. $6 \times 35 = 210$

5. $7 \times 45 = 315$

6. $7 \times 13 = 91$

7. $3 \times 1421 = 4263$

8. $5 \times 1213 = 6065$

9. $6 \times 1314 = 7884$

10. $9 \times 333 = 2997$

5 QUICK QUESTIONS

Use any strategy but not a calculator.

1. $-2 + -3$
$= -5$

2. -2×-3
$= 6$

3. $\frac{2}{5} + \frac{1}{5}$
$= \frac{3}{5}$

4. $\frac{2}{3} \times \frac{1}{5}$
$= \frac{2}{15}$

5. $\frac{2}{3} \div \frac{1}{5}$
$= \frac{10}{3}$

REMEMBER LAST YEAR'S WORK

Which is the most sensible statement? Choose from the 4 alternatives.

1. Keith's 20 year old brother is (1.2), (1.8), (2.6) or (6) metres tall.

    1.8

2. Samantha's baby sister weighs (30 gms), (300 gms), (3 kg) or (30 kg)?
   Her older sister weighs (30 gms), (300 gms), (3 kg) or (30 kg)?

    3kg, 30kg

3. Wayne's car averages 9.5 litres of fuel for every 100 km travelled. On his holiday he expects to drive 700 km and estimates he will need about ($30), ($60), ($90), ($120) for petrol which costs $1.80 per litre.

    $120

Calculate the answers to the following:

4. George sets off on a training ride at 1.50pm and cycles for 4½ hours. What time did he stop?

    6.20pm

5. A plane is due to leave Wellington Airport at 20:50 and arrive at its destination at 03:05 the next day. It actually arrives 45 minutes early.
   At what time does it arrive?

    2.20am

NUMBER APPLICATIONS

1. A team in the Australian soccer league has 17 points. They have played 15 games. For a win there is 3 points, for a draw there is 1 point.

   What are the various combinations (of Win, Draw or Lose) that would have summed to 17?

    5 wins + 2 draws + 8 losses

    4 wins + 5 draws + 6 losses

    3 wins + 8 draws + 4 losses

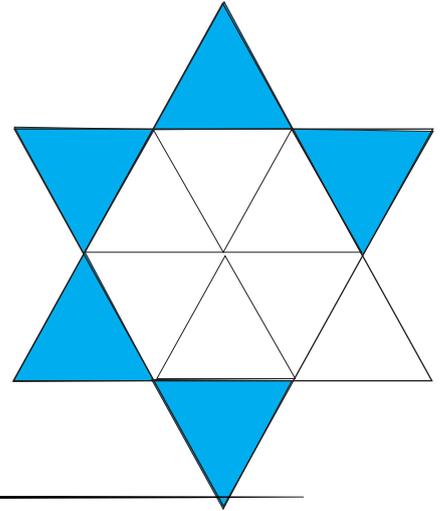   *2 wins + 11 draws + 2 losses*

    1 win + 14 draws

BLACK Worksheet 2

INVESTIGATION
The shape on the right is made from two same sized equilateral triangles. What fraction is shaded?

$$\frac{5}{12}$$

ESTIMATION
Estimate the answers to the following. No calculators.

1. 2.1×7 $2 \times 7 = 14$

2. 14.9×10 $15 \times 10 = 150$

3. 294×3 $300 \times 3 = 900$

4. 13.9×5 $14 \times 5 = 70$

5. 1.93×1.1 $2 \times 1 = 2$

6. 2.73×3.9 $3 \times 4 = 12$

7. $699 \div 6.8$ $700 \div 7 = 100$

8. $15.31 \div 4.9$ $15 \div 5 = 3$

9. $19973 \div 5.3$ $20000 \div 5 = 4000$

10. $15.2 \div 5.3$ $15 \div 5 = 3$

FRACTIONS
Add and subtract the following fractions.

1. $\dfrac{1}{7} + \dfrac{3}{7} = \dfrac{4}{7}$

2. $\dfrac{4}{11} - \dfrac{2}{11} = \dfrac{2}{11}$

3. $\dfrac{2}{3} + \dfrac{4}{5} = \dfrac{22}{15}$

4. $\dfrac{5}{7} - \dfrac{2}{6} = \dfrac{16}{42} = \dfrac{8}{21}$

5. $\dfrac{3}{5} - \dfrac{1}{3} = \dfrac{4}{15}$

6. $\dfrac{15}{6} - 1 = \dfrac{9}{6} = \dfrac{3}{2} = 1\dfrac{1}{2}$

7. $2\dfrac{1}{4} + \dfrac{1}{3} = 2\dfrac{7}{12}$

8. $1\dfrac{3}{4} + 1\dfrac{1}{5} = 2\dfrac{19}{20}$

9. $\dfrac{1}{2} - \dfrac{2}{5} = \dfrac{1}{10}$

10. $\dfrac{2}{5} + \dfrac{1}{7} = \dfrac{19}{35}$

NUMERACY STRATEGIES
Division by multiplication. Solve these by rewriting as multiplication.

e.g. $45 \div 9 \longrightarrow 9 \times \boxed{5} = 45$

1. $32 \div 8$ $8 \times 4 = 32$

2. $63 \div 7$ $7 \times 9 = 63$

3. $120 \div 12$ $12 \times 10 = 120$

4. $84 \div 4$ $4 \times 21 = 84$

5. $96 \div 8$ $8 \times 12 = 96$

6. $112 \div 4$ $4 \times 28 = 112$

7. $144 \div 4$ $4 \times 36 = 144$

8. $92 \div 4$ $4 \times 23 = 92$

9. $315 \div 5$ $5 \times 63 = 315$

10. $159 \div 3$ $3 \times 53 = 159$

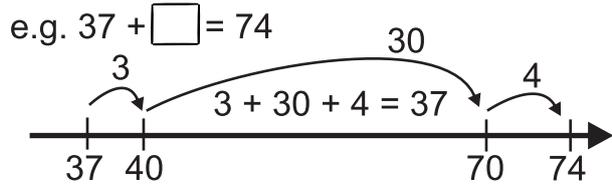|		
2.00	0.25	1.50
0.75	1.25	1.75
1.0	2.25	0.5

PUZZLE - MAGIC SQUARE
A magic square has all columns, rows and diagonals that sum to the same number.

Find the missing numbers to make a magic square.

NUMERACY STRATEGIES
Addition by use of a number line.

e.g. 37 + ☐ = 74

$$3 + 30 + 4 = 37$$

37 40   70 74

1.   17 + | **35** | = 52

2.   25 + | **56** | = 81

3.   37 + | **56** | = 93

4.   42 + | **69** | = 111

5.   53 + | **78** | = 131

6.   42 + | **139** | = 181

7.   121 + | **42** | = 163

8.   117 + | **24** | = 141

9.   121 + | **55** | = 176

10.   119 + | **38** | = 157

QUICK QUESTIONS
Use any strategy but do not use a calculator.

1.   179 + 93 =   **272**

2.   179 - 93 =   **86**

3.   312 × 7 =   **2184**

4.   Convert 0.17 to a percentage.

     17%

5.   What is 10% of 80?

     8

6.   -2 + 5 =   **3**

7.   -2 × 5 =   **-10**

8. $1\frac{1}{4} + 2\frac{1}{2} =$ $\mathbf{3\frac{3}{4}}$

9. $\frac{3}{7} + \frac{4}{10} =$ $\mathbf{\frac{58}{70} = \frac{29}{35}}$

10.   31.5 × 10 =   **315**

TESTING TIMES
Two companies offer Brad a job. Both offer $40,000 starting salary. For the first two years, Company 1 will give him a pay rise of $6000 at the end of each year. For the first two years, Company 2 will give him a rise of $1500 every six months. Which deal is the best?

C1: $40,000 + $46,000

Total = $86,000

C2: $20,000 + $21,500

+ $23,000 + $24,500

Total = $89,000

NUMBER SKILLS
Work out the answers.

1. 40 × 50 = __2000__

2. 0.8 × 0.9 = __0.72__

3. 30 × 0.7 = __21__

4. 0.6 × 4 = __2.4__

5. 4.2 ÷ 0.7 = __6__

6. Divide 1720 by 8   __215__

7. Divide 1720 by 0.8   __2150__

8. What is the square root of 0.25? __0.5__

9. Simplify 0.1 × 0.2 × 0.2 __0.004__

10. Write $\frac{5}{8}$ as a decimal. __0.625__

11. 123 + [__855__] = 978

12. [__17__] × 26 = 442

13. [__3149__] - 184 = 2965

14. [__69965__] ÷ 35 = 1999

15. 50 × 50 + [__50__] = 2550

CROSS NUMBER

1. 1	5	3	2. 8		3. 7
9			0		7
4. 2	9	5. 5		6. 9	0
		7. 1	5		
8. 2	1	9. 3		10. 7	0
1		11. 1	7	2	

Across

1. $\frac{2}{5}$ of 3845
4. 118 × 2.5
6. $\frac{3}{7}$ of 210
7. $4\frac{1}{2} \times 3\frac{1}{3}$
8. $\frac{3}{4}$ of 284
10. Write $\frac{21}{30}$ as a %
11. $\frac{4}{7}$ of 301

Down

1. 80% of 240
2. Convert $\frac{4}{5}$ to a percent
3. 2310 ÷ 3
5. Convert 5.131×10^3 to ordinary form
8. Increase 15 by 40%
9. $\frac{1}{3}$ of 93
10. Reduce 80 by 10%

BLACK Worksheet 3

APPLICATION

John pays one third of his salary to his parents for board. Mary gives three eighths of her salary for board. Who pays the most board if John earns $1230 a week and Mary earns $1120 a week?

$$\$1230 \times \frac{1}{3} = \$410$$

$$\$1120 \times \frac{3}{8} = \$420 \quad \text{Mary does}$$

FRACTION PROBLEMS

1. $\frac{1}{2} \times 5 = \dfrac{5}{2}$

2. $\frac{2}{3} \times 6 = \dfrac{12}{3}$

3. $\frac{1}{3} \times \frac{1}{4} = \dfrac{1}{12}$

4. $\frac{2}{3} \times \frac{2}{5} = \dfrac{4}{15}$

5. $21 \times \frac{1}{3} = \dfrac{21}{3} = 7$

6. $\frac{2}{3} \times \frac{4}{5} = \dfrac{8}{15}$

7. $\frac{1}{5} \times \frac{2}{7} = \dfrac{2}{35}$

8. $1\frac{1}{2} \times 3\frac{1}{4} = \dfrac{39}{8} = 4\frac{7}{8}$

9. $1\frac{1}{5} \times 1\frac{1}{5} = \dfrac{36}{25} = 1\frac{11}{25}$

10. $\frac{3}{7} \times \frac{7}{3} = \dfrac{21}{21} = 1$

11. $8 \div \frac{1}{2} = 16$

12. $8 \div \frac{1}{4} = 32$

13. $8 \div \frac{1}{1000} = 8000$

14. $8 \div 0 = \text{undefined}$

15. $\frac{1}{2} \div \frac{1}{4} = 2$

16. $\frac{2}{3} \div \frac{4}{5} = \dfrac{5}{6}$

17. $\frac{7}{11} \div 2 = \dfrac{7}{22}$

18. $1\frac{1}{4} \div 2\frac{1}{2} = \dfrac{1}{2}$

19. $2\frac{3}{5} \div 1\frac{1}{4} = 2\frac{2}{25}$

20. $6\frac{1}{3} \div 3\frac{1}{6} = 2$

PUZZLE

Chloe left a large sum of money in her will to her family. Jim received one half of the total sum, Joan received one third of what was left and Mary received one quarter of what was left after that. There was then $5 000 left over which was split amongst the remaining family members. What was the original sum of money left in Chloe's will?

$$Jim = \frac{1}{2} \qquad Joan = \frac{1}{6} \qquad Mary = \frac{1}{12} \qquad Total = \frac{3}{4}$$

Total of will was $20,000

NUMERACY STRATEGIES (SUBTRACTION)

Subtraction sums can be rewritten as addition: e.g.   73 - 29
This can be rewritten to: 29 + $\boxed{44}$ = 73

Rewrite these as addition sums.

1.   47 - 9

   9 + 28 = 47

6.   111 - 93

   93 + 18 = 111

2.   38 - 15

   15 + 23 = 38

7.   125 - 89

   89 + 36 = 125

3.   73 - 25

   25 + 48 = 73

8.   136 - 73

   73 + 63 = 136

4.   93 - 47

   47 + 46 = 93

9.   152 - 118

   118 + 34 = 152

5.   88 - 59

   59 + 29 = 88

10.   167 - 139

   139 + 28 = 167

NUMERACY STRATEGIES (MULTIPLICATION)

To make it easier multiplying you can halve one number and double the other.
e.g. $6 \times 22 \longrightarrow 3 \times 44 = 132$
          or $12 \times 11 = 132$

You could also treble one number and find one third of the other
e.g. $1.5 \times 6 \longrightarrow 4.5 \times 2 = 9$
          or $0.5 \times 18 = 9$

Use a strategy to calculate these products. Show your working

1. 3×16 $= 48$

2. 28×2 $= 56$

3. 4.6×4 $= 18.4$

4. 1.6×4 $= 6.4$

5. 1.3×4 $= 5.2$

6. 2.7×2 $= 5.4$

7. 1.2×3 $= 3.6$

8. 2.4×3 $= 7.2$

9. 0.9×2 $= 1.8$

10. 1.5×4 $= 6$

TEN QUICK QUESTIONS

Use any strategy but not a calculator.

1. $223 + 89 =$ ___312___

2. $221 - 195 =$ ___26___

3. List the first 6 multiples of 3.
   ___3, 6, 9, 12, 15, 18___

4. What are the factors of 32?
   ___1, 2, 4, 8, 16___

5. $-2 - (-3) =$ ___1___

6. $-312 + (-123) =$ ___435___

7. $-3 + -2 \times -4 =$ ___5___

8. $(-3)^2 =$ ___9___

9. $2.31 \times 100 =$ ___231___

10. $7.2 \times 0.7 =$ ___5.04___

NUMBER SKILLS
Find the values of:

1. $2^4 =$ ___16___

2. $8^3 =$ ___512___

3. $10^5 =$ ___100,000___

4. $2^3 \times 3^2 =$ ___72___

5. $2^2 \times 5 =$ ___20___

6. $\sqrt{121} =$ ___11___

7. $\sqrt{64} =$ ___8___

Write in index form:

8. $3^2 \times 3 =$ ___3^3___

9. $2^{10} \div 2^5 =$ ___2^5___

10. $(2^2)^2 =$ ___2^4___

PERCENTAGES

1. Express 36% as a fraction.
   $$\frac{36}{100} = \frac{9}{25}$$

2. Express 22.5% as a decimal.
   0.225

3. Find 48% of 3 metres
   1.44 m (or 144 cm)

4. Increase $20 by 15%
   $23

5. Decrease $80 by 45%
   $44

6. Express $5\frac{1}{2}$ % as a fraction.
   $$\frac{11}{200}$$

7. Express $12\frac{1}{4}$ % as a decimal.
   0.1225

8. Write $\frac{5}{8}$ as a percentage.
   62.5%

9. Increase 300 by 5%
   315

10. Decrease $120 by 12.5%
    $105

BLACK Worksheet 4

APPLICATION

A block of gold that weighs 45 kg is cut into 4 pieces. Each piece is twice as heavy as the preceding one. How much does each of the four pieces weigh?

$x + 2x + 4x + 8x = 45, x = 15$

Weights are 3kg, 6kg, 12kg, 24kg

FRACTIONS AND DECIMALS

Convert these fractions to their simplest form.

1. $0.2 = \dfrac{1}{5}$

2. $0.5 = \dfrac{1}{2}$

3. $0.05 = \dfrac{1}{20}$

4. $0.005 = \dfrac{1}{200}$

5. $0.15 = \dfrac{3}{20}$

6. $0.35 = \dfrac{7}{20}$

7. $0.176 = \dfrac{22}{125}$

8. $0.319 = \dfrac{319}{1000}$

9. $0.42 = \dfrac{21}{50}$

10. $0.120 = \dfrac{3}{250}$

DECIMALS

Add and subtract the following:

1. $4.25 + 1.11 = 5.36$

2. $3.72 + 2.61 = 6.33$

3. $4.18 + 2.97 = 7.15$

4. $1.43 + 1.39 = 2.82$

5. $0.176 + 0.188 = 0.364$

6. $4.25 - 1.11 = 3.14$

7. $3.72 - 2.16 = 1.56$

8. $4.18 - 2.97 = 1.21$

9. $0.188 - 0.176 = 0.012$

10. $1.43 - 1.39 = 0.04$

APPLICATION

If John takes one third of a cake and Mary takes two fifths of what is left, how much is left for Michael?

$$\frac{6}{15}$$

PUZZLES

How many rectangles can be formed within these dots so that the vertices of each rectangle are 4 of the dots?
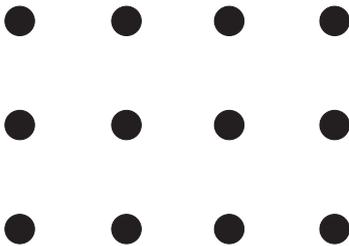
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The diagram below shows an 8 × 8 square grid
How many squares of any size are there?

203
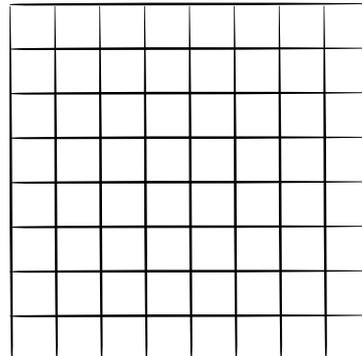
QUICK QUESTIONS
Use any strategy but not a calculator.

1. 149 + 53 = __202__

2. 173 - 89 = __84__

3. Simplify $\frac{8}{12}$ __$\frac{2}{3}$__

4. $\frac{2}{3} \times \frac{4}{7}$ = __$\frac{8}{21}$__

5. $\frac{2}{3} \div 4$ = __$\frac{2}{12} = \frac{1}{6}$__

6. $\frac{11}{17} - \frac{3}{17}$ = __$\frac{8}{17}$__

7. $\frac{3}{5} - \frac{1}{4}$ = __$\frac{7}{20}$__

8. -2^2 = __-4__

9. 1.69 × 10 = __16.9__

10. 3.6 × 0.4 = __1.44__

NUMERACY STRATEGIES
Additions to a power of 10.

e.g. $74 + 96$
$$\begin{array}{cc} -4 & +4 \\ \hline 70 & + 100 \end{array} = 170$$

+ 4   + 96

70   74   170

+ 100
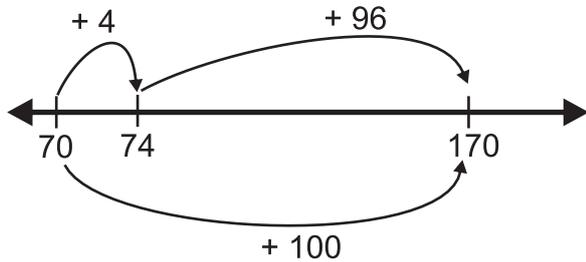
1. 73 + 94 = *167*

2. 15 + 89 = *104*

3. 131 + 93 = *224*

4. 155 + 49 = *204*

5. 169 + 87 = *256*

6. 63 + 97 = *160*

7. 77 + 98 = *175*

8. 123 + 97 = *220*

9. 243 + 197 = *440*

10. 345 + 293 = *638*

NUMERACY STRATEGIES
Subtracting to a power of 10.

e.g. $135 - 96$
$$\begin{array}{cc} +4 & +4 \\ \hline 139 & - 100 \end{array} = 39$$

- 96

39   135 139

- 100
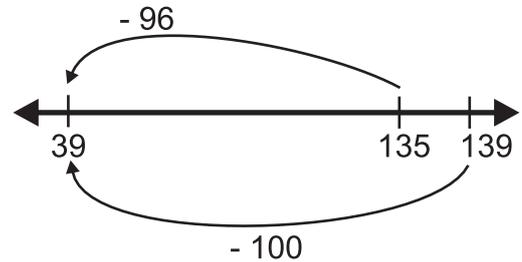
1. 127 - 97 = *30*

2. 235 - 193 = *42*

3. 175 - 89 = *86*

4. 253 - 96 = *157*

5. 262 - 49 = *213*

6. 111 - 96 = *15*

7. 176 - 88 = *88*

8. 183 - 89 = *94*

9. 539 - 97 = *442*

10. 1126 - 997 = *129*

WHAT IS FAT?

a. Calculate the answer and find it below.

b. When completed, cross out all those fractions that were not used.

c. Starting from the top line, write all the letters that remain to answer the question.

1. $\frac{5}{8} - \frac{1}{4} = \frac{3}{8}$

2. $\frac{5}{6} + \frac{1}{3} = 1\frac{1}{6}$

3. $\frac{3}{4} - \frac{1}{3} = \frac{5}{12}$

4. $\frac{2}{3} + \frac{3}{5} = 1\frac{4}{15}$

5. $\frac{4}{5} - \frac{1}{4} = \frac{11}{20}$

6. $\frac{1}{3} + \frac{3}{8} = \frac{17}{24}$

7. $\frac{5}{6} - \frac{1}{4} = \frac{7}{12}$

8. $\frac{3}{10} + \frac{3}{4} = 1\frac{1}{20}$

9. $\frac{1}{5} + \frac{5}{6} = 1\frac{1}{30}$

10. $\frac{7}{9} - \frac{1}{6} = \frac{33}{54} = \frac{11}{18}$

$\frac{3}{7}$	$\frac{3}{4}$	$1\frac{1}{30}$	$\frac{5}{6}$	$1\frac{2}{40}$	$1\frac{1}{6}$	$\frac{3}{8}$	$\frac{12}{16}$	$\frac{7}{10}$	$\frac{3}{7}$
F	C	S	H	I	T	O	P	J	F

$\frac{6}{13}$	$\frac{17}{24}$	$\frac{7}{10}$	$1\frac{1}{20}$	$\frac{5}{6}$	$1\frac{4}{15}$	$\frac{33}{54}$	$1\frac{1}{20}$	$1\frac{1}{3}$	$\frac{15}{24}$
W	R	J	E	H	D	U	E	Q	X

$\frac{7}{12}$	$\frac{7}{9}$	$\frac{7}{13}$	$1\frac{1}{20}$	$\frac{1}{8}$	$\frac{35}{54}$	$\frac{33}{54}$	$\frac{5}{12}$	$\frac{5}{6}$	$\frac{11}{20}$
N	V	K	E	B	U	R	G	H	Y

STORED ENERGY

SEQUENCES

Look at the sequence on the right:

Find the difference between consecutive terms:

Find the difference between these numbers.

	2		5		10		17		26
		3		5		7		9	
			2		2		2		

Assume that the 3rd row continues as a row of 2s. You can then calculate write the 2nd and top rows. Use this method to continue the sequences below:

1. 1, 3, 7, 13, 21, ...31..., ...43..., ...57...

2. 2, 7, 13, 20, 28, ...37..., ...47..., ...58...

3. 5, 10, 20, 35, 55, ...80..., ...110..., ...145...

BLACK Worksheet 5

APPLICATION

Mary receives a rise and her salary increases from $35,000 to $37,500.
John receives a rise and her salary increases from $34,000 to $36,500.
Who has received the biggest percentage increase?   *John*

Mary: $2500 ÷ $35000 = 0.071 (7.1% increase)

John: $2500 ÷ $34000 = 0.073 ((7.3% increase)

NUMERACY STRATEGIES

Multiplication and division by using knowledge of place value.

e.g.   **2·4 × 6**   = 2 × 6 + 0.4 × 6   **3·6 ÷ 2**   = 3 ÷ 2 + 0.6 ÷ 2
                     = 12   + 2.4   = 1.5   + 0.3
                     = 14.4   = 1.8

1.   2.2 × 2 = *4.4*   6.   7.5 ÷ 3 = *2.5*

2.   3.5 × 4 = *14*   7.   12.6 ÷ 7 = *1.8*

3.   5 × 3.4 = *17*   8.   10.2 ÷ 2 = *5.1*

4.   23 × 0.4 = *9.2*   9.   2.68 ÷ 2 = *1.34*

5.   2.16 × 3 = *6.48*   10.   3.04 ÷ 0.04 = *76*

CROSS NUMBER

1.3	8	2.4		3.3	4.5
8		2			0
5.2	5	2		6.5	6
		7.4	8.3	5	
9.5	2		0		10.4
6		11.2	4		5

Across

1. Increase 320 by 20%

3. Convert $\frac{7}{20}$ to a percentage.

5. Find 60% of 420

6. Increase 50 by 12%

7. Find 75% of 580

9. Find 65% of 80

11. Find 60% of 40

Down

1. Find 40% of 955

2. Increase 3840 by 10%

4. Increase 440 by 15%

6. Find 250% of 22

8. Reduce 380 by 20%

9. Reduce 70 by 20%

10. Find 60% of 75

PUZZLE
How many triangles can you see in the large square?
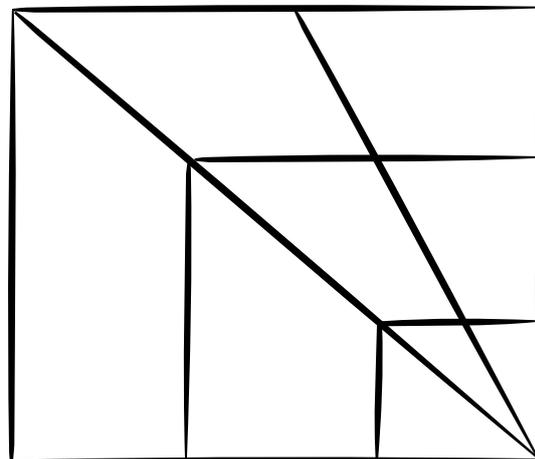
12 triangles

NUMERACY STRATEGIES

Integer addition: $3 + {}^-4$ =

= -1

${}^-2 + {}^-3$ =

= -5

1. 2 + -3 = -1

2. 5 + -2 = 3

3. 9 + -3 = 6

4. 21 + -8 = 13

5. 48 + -21 = 27

6. -3 + -2 = -5

7. -5 + -3 = -8

8. -11 + -3 = -14

9. -21 + -7 = -28

10. -31 + -22 = -53

DECIMALS
Multiply and divide the following decimals:

1. $2.1 \times 3 =$ 6.3
2. $1.4 \div 7 =$ 0.2
3. $1.5 \times 12 =$ 18
4. $1.52 \times 0.4 =$ 0.608
5. $1.52 \div 0.04 =$ 38

6. $1.52 \times 0.14 =$ 0.2128
7. $6.42 \times 0.01 =$ 0.0642
8. $6.42 \div 0.0001 =$ 64200
9. $84.35 \times 0.01 =$ 0.8435
10. $84.35 \div 0.0001 =$ 843500

INDICES
Simplify the following and leave in index form:

1. $2^3 \times 2^5 =$ 2^8
2. $5^4 \times 5^3 =$ 5^7
3. $13^1 \times 13^2 =$ 13^3
4. $14^3 \times 14^2 =$ 14^5
5. $2^3 \times 2^4 \times 2^7 =$ 2^{14}

6. $2^6 \times 2^7 =$ 2^{13}
7. $8^7 \div 8^3 =$ 8^4
8. $2^7 \div 2^3 =$ 2^4
9. $2^{13} \div (2^8 \times 2^2) =$ 2^3
10. $2^6 \times 2^4 \div 2^3 =$ 2^7

NUMERACY STRATEGIES - STANDARD FORM

Write these numbers in scientific form:

1. $300 =$ 3.0×10^2
2. $215 =$ 2.15×10^2
3. $0.05 =$ 5.0×10^{-2}
4. $0.0013 =$ 1.3×10^{-3}
5. $0.0000715 =$ 7.15×10^{-5}

Write these numbers in ordinary form:

1. $3.1 \times 10^2 =$ 310
2. $1.65 \times 10^3 =$ 1650
3. $5 \times 10^6 =$ 5000000
4. $2.3 \times 10^{-3} =$ 0.0023
5. $2.0 \times 10^{-4} =$ 0.0002

27	17	21	30
28	18	22	31
15	5	9	18
16	6	10	19

Any combination of numbers will work!

MAGIC MATRIX

Choose any 4 numbers such that exactly one number is selected from each row and exactly one number is selected from each column. Sum together your chosen numbers and check the result.

e.g. $27 + 5 + 10 + 31 = 73$

TWENTY QUICK QUESTIONS

Use any strategy but not a calculator.

1. 24 × 3 = _72_

2. 128 ÷ 4 = _32_

3. -4 × -3 = _12_

4. -18 - 22 = _-40_

5. $\frac{3}{4} \times \frac{4}{5}$ = _$\frac{12}{20}$ or $\frac{3}{5}$_

6. $\frac{3}{4} \div \frac{4}{5}$ = _$\frac{15}{16}$_

7. $\frac{3}{4} + \frac{4}{5}$ = _$\frac{31}{20}$ or $1\frac{11}{20}$_

8. 1.93 × 100 = _193_

9. Convert $\frac{3}{8}$ to a percentage _37.5%_

10. Calculate 75% of $530 _$397.50_

11. 82 × 5 = _410_

12. 4321 × 3 = _12963_

13. -5 × 6 = _-30_

14. -2 + -3 + -4 = _-9_

15. 29.17 ÷ 10 = _2.917_

16. $\frac{2}{3} \times \frac{7}{8}$ = _$\frac{14}{24}$ or $\frac{7}{12}$_

17. Convert 0.03 to a percentage _3%_

18. Calculate 20% of $170 _$34_

19. Increase 350 by 15% _402.5_

20. Reduce $120 by 15% _102_

Two MAGIC MATRIX
Choose any 4 numbers
~~such that exactly one~~

Any combination of numbers will work!

numbers and check the result.

17	⑬	25	20
8	4	16	⑪
㉗	23	35	30
12	8	⑳	15

27
13
20
11

+ _71_

1	2	3	④
⑤	6	7	8
9	10	⑪	12
13	⑭	15	16

5
14
11
4

+ _34_

BLACK Worksheet 6

APPLICATION
Brad scored 63 out of 90 in Test 1
He then scored 57 out of 80 in Test 2.
Did he improve? Calculate the percentage result in both.

Test 1 = 70%

Test 2 = 71.25% – YES an improvement!

STANDARD FORM (1)
Convert the following into standard form.

1. 21.3 2.13×10^{1}
2. 0.214 2.14×10^{-1}
3. 0.0056 5.6×10^{-3}
4. 234.1 2.341×10^{2}
5. 0.00073 7.3×10^{-4}

STANDARD FORM (2)
Convert the following from standard form to ordinary numbers:

1. 1.73×10^{1} 17.3
2. 2.03×10^{-3} 0.00203
3. 1.75×10^{2} 175
4. 2.829×10^{-3} 0.002829
5. 1.83×10^{3} 1830

INTEGERS
Evaluate the following integer problems.

1. (+2) + (+3) = 5
2. (+2) - (-3) = 5
3. (-2) - (-3) = 1
4. (-2) × (-3) = 6
5. (-2) × (+3) = −6

6. (-20) ÷ 5 = −4
7. (-24) ÷ (-4) = 6
8. (-2) + 6 ÷ (-2) + 3 = −2
9. 5 - (-3) × (-2) - (-5) = 4
10. 5 × [2 × (-3) + (-1)] = −35

PUZZLE

There are 16 numbers each within a certain shape. Place them in the grid so that each line of 4 has 4 different numbers and shapes.
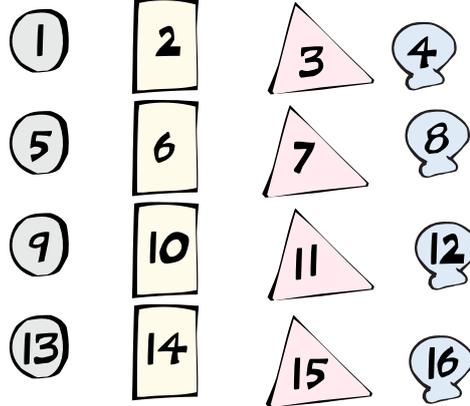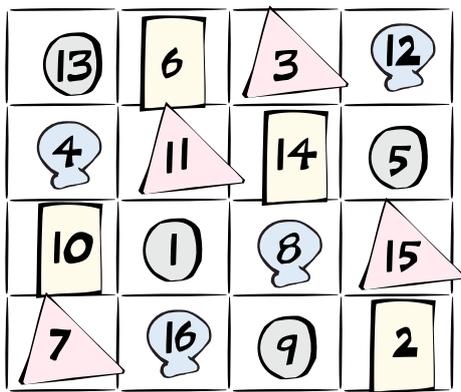
NUMERACY STRATEGIES - INTEGERS

Subtraction $\quad$ -3 - -2 = -1

$-2 + +4 = 2$

$-2 - +4 = -6$
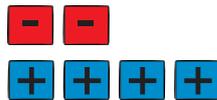
1. $\quad$ -2 - -1 = $\quad$ _-1_
2. $\quad$ -2 - -3 = $\quad$ _1_
3. $\quad$ -7 - -3 = $\quad$ _-4_
4. $\quad$ -2 - +3 = $\quad$ _-5_
5. $\quad$ -5 - +7 = $\quad$ _-12_
6. $\quad$ -3 - -5 = $\quad$ _2_
7. $\quad$ -11 - +3 = $\quad$ _-14_
8. $\quad$ -17 - -6 = $\quad$ _-11_
9. $\quad$ -17 - +8 = $\quad$ _-25_
10. $\quad$ -21 - +25 = $\quad$ _-46_

NUMERACY PRACTICE

Put a +, -, × or ÷ sign into each box to make the equations correct.

1. $\quad$ 5 $\boxed{+}$ (3 + 5) = 13

2. $\quad$ $9^2 - 9^2 = 8 \boxed{-} 8$

3. $\quad$ $\sqrt{100} \boxed{\div} (2 \times 5) = 1$

4. $\quad$ $2^4 + 2^3 = 8 \boxed{\times} 3$

5. $\quad$ 36 $\boxed{+}$ (16 × 3) = 84

NUMERACY STRATEGIES - HIGHEST COMMON FACTORS

The largest common factor of two or more numbers is called the highest common factor. e.g Find the highest common factor of 24 and 30.

$$24 \quad = 6 \times 4$$
$$= 3 \times 2 \times 2 \times 2$$
$$= 3 \times 4 \times 2$$
$$= 3 \times 2 \times 2 \times 2$$

$$30 \quad = 3 \times 10$$
$$= 3 \times 2 \times 5$$
$$= 6 \times 5$$

Factors of 24 are 2, 3, 4, 6, 8
Factors of 30 are: 2, 3, 5, 6, 10

Common factors are: 2, 3, 6   Highest Common Factor = 6

Find the highest common factors of the following pairs of numbers:

1.	30, 40	*10*
2.	56, 64	*8*
3.	72, 40	*8*
4.	84, 46	*2*
5.	112, 140	*28*

6.	325, 245	*5*
7.	300, 280	*20*
8.	110, 160	*10*
9.	320, 180	*20*
10.	64, 84	*4*

QUICK QUESTIONS
Use any strategy but not a calculator

1. 179
   + 81
   ———
   260

2. 312
   × 7
   ———
   2184

3. -3 × -4 = *12*

4. -2 - (-3) - (-4) = *5*

5. Convert $\frac{5}{8}$ to a percentage *62.5%*

FIFTEEN QUICK QUESTIONS

Use any strategy but not a calculator.

1. $3 \times 4 - (-2) \times 3 =$ _____ 18

2. $1\frac{1}{4} + 2\frac{3}{8} =$ _____ $3\frac{5}{8}$

3. $29.15 \times 10 =$ _____ 291.5

4. Round 31.65 to 1 decimal place 31.7

5. Convert 36% to a decimal _____ 0.36

6. Find 22% of $620 _____ $\$136.40$

7. Find 30% of $315 _____ $\$94.50$

8. Decrease $400 by 12.5% $\$350$

9. Decrease $410 by 70% $\$123$

10. Simplify the ratio 30:45 _____ $2:3$

11. Simplify 3km : 2500m _____ $6:5$

12. Split 350 into a ratio of 2:3 _____ $140, 210$

13. Bill earns $67.50 in 3 hours.
    How much does he earn in 40 hours?
    _____ $\$900$

14. $\sqrt{400}$ _____ 20

15. Simplify $\frac{140}{180}$ _____ $\frac{7}{9}$

ESTIMATING AND ROUNDING OFF

Round the numbers on the number line below to the nearest 100.

```
|--|--|-------|----|---|--|---|----|
  100    200     300       400
```

1. 120 is approximately _____ 100

2. 260 is approximately _____ 300

3. 350 is approximately _____ 400

Two MAGIC MATRIX

Choose any 4 numbers

Any combination of numbers will work!

such that exactly one

numbers and check the result.

27	17	21	(30)
(28)	18	22	31
15	(5)	9	18
16	6	(10)	19

```
| 28 |
|  5 |
| 10 |
| 30 |
+
  73
```

4	(5)	6	7
8	9	(10)	11
(12)	13	14	15
16	17	18	(19)

```
| 12 |
|  5 |
| 10 |
| 19 |
+
  46
```

BLACK Worksheet 7
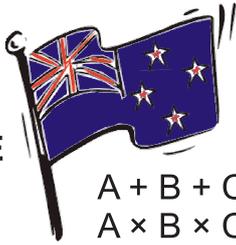
PUZZLE

A + B + C = 15
A × B × C = 120

Find the value of A, B, C.

$$A = 4, B = 5, C = 6$$

-39

SUBSTITUTION (1)

Evaluate the following expressions by substituting the following values:
a = 6, b = 3, c = -2

1.	$3a$	18
2.	$5b$	15
3.	$2a - 2b$	6
4.	$3c - b$	-9
5.	$2(a + b)$	18
6.	$\frac{a + b}{c}$	-4.5
7.	$-3(a - b)$	-9
8.	$\frac{3ab}{c}$	-27
9.	$2(a + b) - c$	20
10.	$\frac{c}{a + b}$	$\frac{-2}{9}$

SUBSTITUTION (2)

Evaluate the following expressions by substituting the following values:
x = 3, y = -5

1.	$x + y$	-2
2.	$3x - 2y$	19
3.	$4(x - y)$	32
4.	$\frac{5x}{y}$	-3
5.	$2xy - x^2$	
6.	$9x + y$	22
7.	$x(x + y)$	-6
8.	$y^2 - x^2$	16
9.	$7xy$	-105
10.	y^2	25

SEQUENCES AND PATTERNS

Complete the tables for each pattern and try and establish the rule.

1.

Number of forks	1	2	3	4	8	10	n
Number of prongs	4	8	12	16	32	40	4n
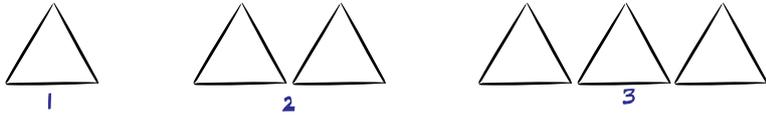
2.

Number of triangles	1	2	3	4	13	29	n
Perimeter	3	6	9	12	39	87	3n
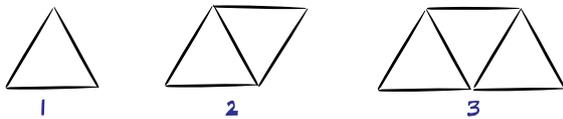
3.

Pattern number	1	2	3	4	20	55	n
Perimeter	3	4	5	6	22	57	n + 2

4.

Pattern number	1	2	3	4	8	30	n
Perimeter	4	6	8	10	18	62	2n + 2
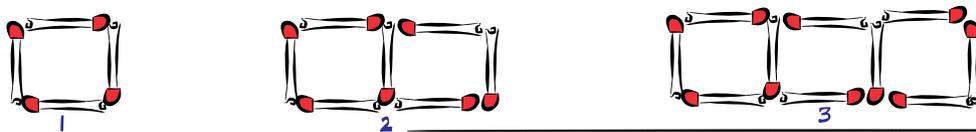
5.

Pattern number	1	2	3	4	8	10	n
Number of matchsticks	4	7	10	13	25	31	3n + 1
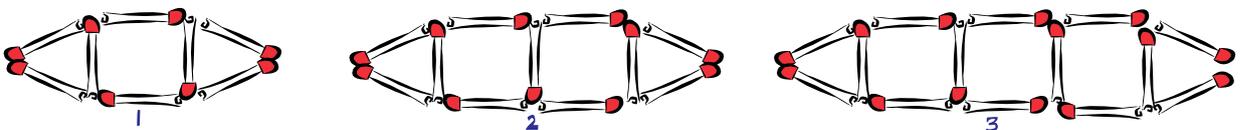
6.

Pattern number	1	2	3	4	7	15	n
Number of matchsticks	8	11	14	17	26	50	3n + 5

TEN QUICK QUESTIONS

1. Simplify $2a + 3a + 4a$ $9a$

2. Simplify $4a + 5b + 2a - 3b$ $6a + 2b$

3. Simplify $x^7 \times x^4 \times x^2$ x^{13}

4. Simplify $\dfrac{x^9}{x^7}$ x^2

5. Work $3a^2$ when $a = 3$

   27

6. Expand $2(x + 3)$

   $2x + 6$

7. Expand and simplify

   $3(x + 2) + 2(x - 4)$

   $5x - 2$

8. What is the value of $2a + 3b$ when $a = 2$ and $b = 4$

   16

9. What is the value of x^2 when $x = -2$

   4

10. Find 3 consecutive numbers that sum to 45

    $14, 15, 16$

INVESTIGATION

Complete the next 3 numbers in each. sequence.

1. $4, 7, 10$ 13 , 16 , 19

2. $1, 4, 9$ 16 , 25 , 36

3. $\dfrac{7}{8}$ $\dfrac{11}{8}$ $\dfrac{15}{8}$ $\dfrac{19}{8}$, $\dfrac{23}{8}$, $\dfrac{27}{8}$

4. $16, 8, 4$ 2 , 1 , $\dfrac{1}{2}$

5. $5, 10, 20$ 40 , 80 , 160

Write down the first 5 terms of these sequences. In each one, the first term is 1.

6. Add 5 each time.

   1 , 6 , 11 , 16 , 21 , 26

7. Subtract 3 each time.

   1 , -2 , -5 , -8 , -11 , -14

8. Multiply by 2 each time.

   1 , 2 , 4 , 8 , 16 , 32

9. Divide by 2 each time.

   1 , $\dfrac{1}{2}$, $\dfrac{1}{4}$, $\dfrac{1}{8}$, $\dfrac{1}{16}$, $\dfrac{1}{32}$

10. Add 1 then 2, 3, 4, ...

    1 , 2 , 4 , 7 , 11 , 16

MISCELLANEOUS PRACTICE

1. What is the smallest number that 5, 6 and 10 all divide exactly? 30

2. If 5 similar books weigh 4.5 kg, what will 3 books weigh? $7.5kg$

3. The area of a triangle is 36 cm^2. If the base is 9 cm what is the height? 8

4. If a car costing \$5000 is sold for \$4000 what was the percentage loss? 20%

5. What is the approximate value of the square root of $(7^2 + 7^2)$ 10

WHAT'S THE DIFFERENCE BETWEEN PLANTS AND ANIMALS?

a.   Factorise the following expressions.
b.   Find the correct letter in the table.
c.   Put the letter in the table above the question number for that letter.

1. $2x + 2$ *A*

2. $2x^2 - 6x$ *M*

3. $x^2 - 5x + 6$ *R*

4. $x^2 - x - 12$ *O*

5. $2x^2 - 18$ *D*

6. $8x - 10$ *N*

7. $10x^2 + 4x$ *L*

8. $x^2 - 16$ *E*

9. $x^2 + 3x - 10$ *T*

10. $2x^2 + 10x + 12$ *C*

11. $x^2 - 2x$ *I*

12. $x^2 + 5x + 6$ *S*

13. $x^2 - 16x$ *F*

14. $x^2 - 6x + 5$ *Y*

Puzzle pieces:

Expression	Letter
$2(x + 2)(x + 3)$	C
$(x - 2)(x - 3)$	R
$4(2x - 5)$	N
$x(x - 16)$	F
$(x - 5)(x + 1)$	Y
$2x(x - 3)$	M
$(x - 4)(x + 4)$	E
$(x + 5)(x - 2)$	T
$2(x + 1)$	A
$(x - 4)(x + 3)$	O
$x(x - 2)$	I
$(x + 2)(x + 3)$	S
$2(x - 3)(x + 3)$	D
$2x(5x + 2)$	L

A	N	I	M	A	L	S	A	R	E
1	6	11	2	1	7	12	1	3	8

F	O	O	T	L	O	O	S	E
13	4	4	9	7	4	4	12	8

A	N	D	F	A	N	C	Y	F	R	E	E
1	6	5	13	1	6	10	14	13	3	8	8

BLACK Worksheet 8

EQUATION SOLVING (1)

1. $2x + 3 = 7$

$x = 2$

2. $2x - 3 = 11$

$x = 7$

3. $5x - 2 = -12$

$x = -2$

4. $-3x - 2 = 7$

$x = -3$

5. $5 - 2x = 9$

$x = -2$

6. $\frac{x}{3} - 1 = 2$

$x = 9$

7. $\frac{x}{2} + 3 = 9$

$x = 12$

8. $\frac{x}{5} - 1 = -3$

$x = -10$

9. $4 - \frac{x}{3} = 2$

$x = 6$

10. $3 - \frac{x}{5} = -4$

$x = 35$

EQUATION SOLVING (2)

1. $2(x + 1) = 12$

$x = 5$

2. $3(x - 2) = 9$

$x = 5$

3. $4(x - 3) = -12$

$x = 0$

4. $2(3 - x) = 6$

$x = 0$

5. $2(x - 1) - x = 5$

$x = 7$

6. $(x + 2) + (x + 3) = 15$

$x = 5$

7. $(x - 1) + (x - 5) = 18$

$x = 2$

8. $2(x - 2) + 3(x - 1) = 3$

$x = 2$

9. $5(x - 1) + 2(x + 1) = 18$

$x = 3$

10. $3(x - 1) = x + 17$

$x = 10$
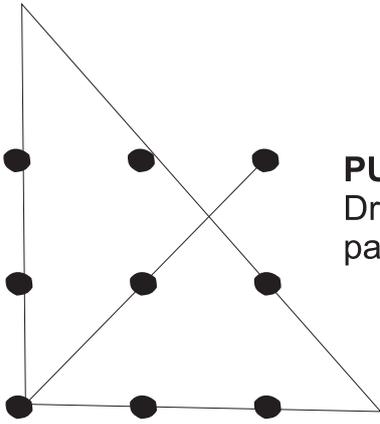
PUZZLE
Draw four straight lines without taking your pen off the paper. Your finished shape should go through all 9 dots.

MAGIC SQUARE
The magic square below uses the numbers 1 to 16.

All the rows, columns and main diagonals add up to 34.

Complete the missing numbers.

16	2	3	13
5	11	10	8
9	7	15	12
4	14	15	1

INVESTIGATION
A man has a bundle of $10 notes in his pocket.

The notes are numbered consecutively from 442426 to 442450.

What is the total value? $250

SEQUENCES AND PATTERNS

Establish a rule for the following patterns.

e.g. 4, 7, 10   If n is the number of the term then n = 1 is the first term.
The rule is: nth term = 3n + 1.

1.   2, 4, 6, 8 $2n$

2.   3, 5, 7, 9 $2n + 1$

3.   1, 4, 16, 25 n^2

4.   2, 5, 17, 26 $n^2 + 1$

Each of the following shapes can be made from sticks. Give the rule for the pattern number and the number of sticks needed to form that shape.
The first one is done for you.
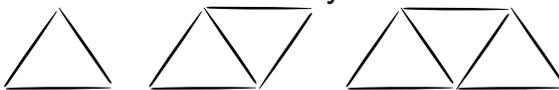
5.   *number of sticks for the nth pattern = 2n + 1*

6. $3n + 1$
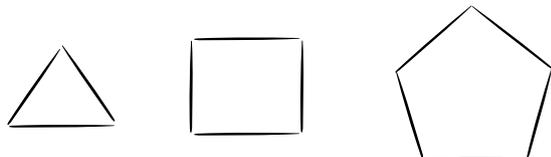
7. $n + 2$
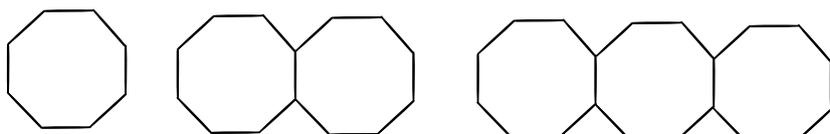
8. $7n + 1$

9.   For the sequence below, give the rule for the number of diagonals.

$n - 1$
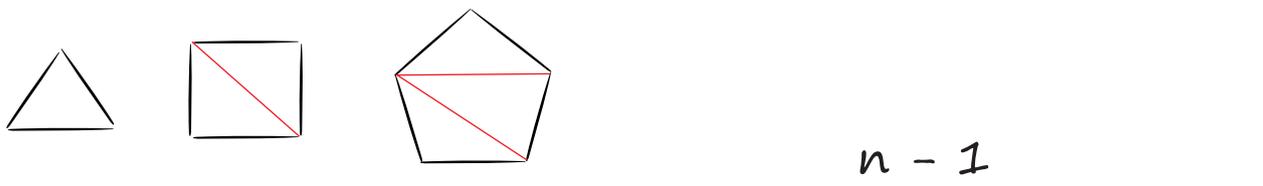
10. For the sequence below, give the rule for the number of white squares across the length.

$n + 2$
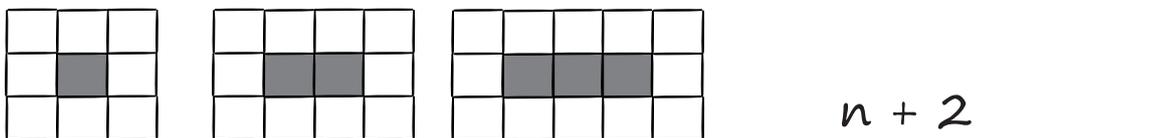
ALGEBRA SKILLS

Simplify the following algebraic expressions.

1. $3c \times 5f$

$\underline{\quad 15cf \quad}$

2. $3x \times -2f =$

$\underline{\quad -6xf \quad}$

3. $-2x \times 3y \times 4r$

$\underline{\quad -24xyr \quad}$

4. $y + 7y \times -4y$

$\underline{\quad -28y^2 + y \quad}$

5. $5x - 4y + 2x - 6y$

$\underline{\quad 7x - 10y \quad}$

6. $4x \times 2x$

$\underline{\quad 8x^2 \quad}$

7. $(5x)^2$

$\underline{\quad 25x^2 \quad}$

8. $(3x^2y^3)^2$

$\underline{\quad 9x^4y^6 \quad}$

9. $\dfrac{16x^2y^4}{10xy^2}$

$\underline{\quad \dfrac{8xy^2}{5} \quad}$

10. $\sqrt{64y^{10}}$

$\underline{\quad 8y^5 \quad}$

FIVE QUICK QUESTIONS

1. Write a rule for the following pattern: 3, 5, 7, 9 . . . .

$\underline{\quad 2n + 1 \quad}$

2. Simplify $x^4 \times x^2 \times x^3$

$\underline{\quad x^9 \quad}$

3. Simplify $6x^4 \times 3x^2$

$\underline{\quad 18x^6 \quad}$

4. Simplify $\dfrac{x^8}{x^4}$

$\underline{\quad x^5 \quad}$

5. Simplify $\dfrac{8x^{11}}{x^4}$

$\underline{\quad 8x^7 \quad}$

PUZZLE

If 1 m^3 of earth weighs 1600 kg, then how much earth would there be in a hole 50 cm × 50 cm × 50 cm?

$\underline{\quad 0.5m \times 0.5m \times 0.5m = 0.125m^3, \text{ i.e. } 200kg \quad}$

BLACK Worksheet 9

ALGEBRA SKILLS
Expand and simplify:

1. $5(x + y)$

$5x + 5y$

2. $x(x + 5)$

$x^2 + 5x$

3. $x(5 - y)$

$5x - xy$

4. $8 + 2(3 - 2x)$

$14 - 4x$

5. $6 - 4(3x - 2)$

$-12x + 14$

6. $5(x - 2) + 3(x - 5)$

$8x - 25$

7. $4(x + 1) + 2(x + 3)$

$6x + 10$

8. $3(x - 1) - 2(x + 1)$

$x - 5$

9. $5(x + 3) - 3(x - 1)$

$2x + 18$

10. $4x + 3(2 - x)$

$x + 6$

ALGEBRA SKILLS (2)
Factorise the following:

1. $3x + 3y$

$3(x + y)$

2. $8x + 20y$

$4(2x + 5y)$

3. $4x + 10y - 6r$

$2(2x + 5y - 3r)$

4. $ax + ay$

$a(x + y)$

5. $10xyz + 22xyr$

$2xy(5z + 11r)$

6. $2x - 2y$

$2(x - y)$

7. $5x - 15y$

$5(x - 3y)$

8. $2x + 6y + 8z$

$2(x + 3y + 4z)$

9. $bx - by$

$b(x - y)$

10. $17xy - 14xyz$

$xy(17 - 14z)$

EQUATION SOLVING
Write an equation for each problem then solve it.

1. A number is doubled, then 3 is subtracted from it to give an answer of 7.
    What was the original number?

    $$2x - 3 = 7$$

    $$x = 5$$

2. A number is divided by 3 then has 4 subtracted from it to give an answer of 8.
    What was the original number?

    $$x \div 3 - 4 = 8$$

    $$x = 36$$

3. 4 less than 5 times a number is 11. What is the number?

    $$5x - 4 = 11$$

    $$x = 3$$

4. 5 is added to a number then multiplied by 7 to give an answer of 56.
    What was the original number?

    $$7(x + 5) = 56$$

    $$x = 3$$

5. Four consecutive numbers add up to 74. What are the numbers?

    $$x + (x + 1) + (x + 2) + (x + 3) = 74$$

    $$17, 18, 19, 20$$

6. The length of a rectangle is 3 metres more than the width. If the perimeter is 22
    metres, find the length and the width.

    $$(x + 3) + x + x + (x + 3) = 22$$

    $$\text{Length} = 7, \text{width} = 4$$

EQUATION SOLVING (continued)

Write an equation for each problem then solve it.

7.   A laptop computer costing $1430 is paid by providing a deposit of $470 and the rest is paid over 12 equal monthly installments.

$$12x + \$470 = \$1430$$

$$x = \$80$$

8.   In a fundraising event participants are given money for each kilometre walked. Jane, Peter and Fred all walk in the event and their total distance walked is 48km. Jane walks 5km more than Peter and Fred walks 5km more than Jane. How much does each person walk?

$$J + P + F = 48, J = P+5, F = J + 5$$

$$Peter = 11km, Jane = 16km, Fred = 21km$$

9.   Peter is three times as old as Michael and the difference of their ages is 48. What are their ages?

$$3x - x = 48$$

$$Peter = 72, Michael = 24$$

10.   Pictured below are two shapes. One is a regular hexagon of side length x + 1 metres and the other a triangle of irregular lengths. Find the value of x if both perimeters are the same.

x + 1

x + 3   x + 4

x + 5
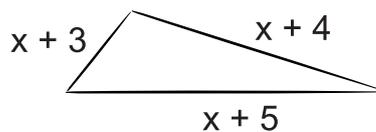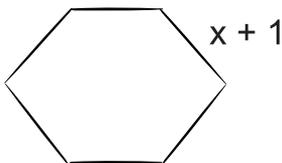
$$6(x+1) = (x+3) + (x+4) + (x+5)$$

$$x = 2$$

SUDOKU

7	5	2	4	9	6	1	8	3
8	9	6	3	2	1	5	7	4
3	1	4	5	7	8	2	9	6
4	6	5	9	3	7	8	1	2
9	3	8	2	1	4	6	5	7
2	7	1	6	8	5	3	4	9
6	8	3	1	4	9	7	2	5
1	2	9	7	5	3	4	6	8
5	4	7	8	6	2	9	3	1

ALGEBRA SKILLS
Factorise the following

1. $2x + 6$

$$2(x + 3)$$

2. $10x - 15$

$$5(2x - 3)$$

3. $x^2 - 4x$

$$x(x - 4)$$

4. $2x^2 - 8x$

$$2x(x - 4)$$

5. $x^2 + 5x + 6$

$$(x + 3)(x + 2)$$

6. $x^2 - 5x + 6$

$$(x - 2)(x - 3)$$

7. $x^2 - x - 12$

$$(x - 4)(x + 3)$$

8. $x^2 - 8x + 7$

$$(x - 7)(x - 1)$$

9. $x^2 - 8x + 12$

$$(x - 6)(x - 2)$$

10. $x^2 - 2x - 15$

$$(x - 5)(x + 3)$$

QUICK QUESTIONS

1. Find the value of T when a = 4, b = 8 and T = 2(a + b). $T = 24$

2. Simplify $6a - 5b - 3a + 3b$ $3a - 2b$

3. Simplify $3a × 4b × 5c$ $60abc$

4. Simplify $5x - 3x^2 - 2x$ $3x - 3x^2$

5. Write a rule for the pattern 2, 4, 6, 8, ... $2n$

6. Write a rule for the pattern 3, 5, 7, 9, ... $2n + 1$

7. Simplify $x^7 × x^2 × x^{-1}$ x^8

8. Simplify $5x^2 × 6x^3$ $30x^5$

9. Simplify $\dfrac{x^{12}}{x^3}$ x^9

10. Simplify $\dfrac{2x^{12}}{x^3}$ $2x^9$

BLACK Worksheet 10

ALGEBRA SKILLS
Expand and simplify:

1. $x(x + 2)$

$$x^2 + 2x$$

2. $(x + 1)(x + 2)$

$$x^2 + 3x + 2$$

3. $(x + 2)(x - 2)$

$$x^2 - 4x$$

4. $(x + 2)(x + 2)$

$$x^2 + 4x + 4$$

5. $(x - 3)(x + 4)$

$$x^2 + x - 12$$

6. $(x - 3)(x - 3)$

$$x^2 - 6x + 9$$

7. $(2x + 1)(3x + 3)$

$$6x^2 + 9x + 3$$

8. $(2x - 1)(3x - 3)$

$$6x^2 - 9x + 3$$

9. $(5x + 2)(4x - 3)$

$$20x^2 - 7x - 6$$

10. $(4x - 2)(3x - 2)$

$$12x^2 - 14x + 4$$

ALGEBRA SKILLS (2)
Simplify the following:

1. $2a + 3a = 5a$

2. $2a + 3a - 4a = a$

3. $5a + 3b + 2a + 4b = 7a + 7b$

4. $5a^2 + 2a + 3a^2 + 4a$

$$= 8x^2 + 6a$$

5. $2a + 7b - a + 3b$

$$= a + 10b$$

6. $2a - 3b - a - 5b$

$$= a - 8b$$

7. $3a^2 - a - 2a^2$

$$= a^2 - a$$

8. $2xy + 4yx$

$$= 6xy$$

9. $3x^2y - 2yx^2 + x^2y - 3yx^2$

$$4x^2y - 5yx^2 = -x^2y$$

10. $4x^2y - 2yx^2$

$$= 2x^2y$$

SOLVING EQUATIONS
Solve the following equations:

1.   x - 3 = 7

   $x = 10$

2.   x + 6 = -11

   $x = -17$

3.   3x = -15

   $x = -5$

4.   -11x = 33

   $x = -3$

5. $\frac{x}{7} = 5$

   $x = 35$

6.   3x - 1 = 8

   $x = 3$

7.   16 - 2x = 19 - 5x

   $x = 1$

8.   8(x - 2) = 24

   $x = 5$

9.   3(x + 2) = 2(x - 5)

   $x = -16$

10. $\frac{3(x - 1)}{2} = -6$

   $x = -3$

ALGEBRA SKILLS
Expand and simplify:

1.   x(x + 3) + 2(x - 4)

   $x^2 + 5x + 8$

2.   x(x - 3) - 2(x + 3)

   $x^2 - 5x - 6$

3.   x(x - 2) + 3(x - 5)

   $x^2 + x - 15$

4.   x(x - 2) + 5(x - 1)

   $x^2 + 3x - 5$

5.   x(x + 7) + 2(x - 1)

   $x^2 + 9x - 2$

6.   2x + 3(x + 1)

   $5x + 3$

7.   7x + x(x - 3)

   $x^2 + 4x$

8.   8x - 2(x + 3)

   $6x - 6$

9.   9x - x(x - 4)

   $-x^2 + 13x$

10.   5x + x(3 - x)

   $8x - x^2$

ARE GENIUSES PERFECT?

Expand and simplify the following expressions.
Write the letter above the answer in the table below.

1. **A** $\quad 3(x + y)$ $\qquad 3x + 3y = A$

2. **P** $\quad 6(x - y)$ $\qquad 6x - 6y = P$

3. **E** $\quad -2(x + y)$ $\qquad -2x - 2y = E$

4. **R** $\quad -3(x - y)$ $\qquad -3x + 3y = R$

5. **S** $\quad 3(x - 3)$ $\qquad 3x - 9 = S$

6. **O** $\quad -2(x + 4)$ $\qquad -2x - 8 = O$

7. **W** $\quad x(3 + x)$ $\qquad 3x + x^2 = W$

8. **H** $\quad x(x - 2)$ $\qquad x^2 - 2x = H$

9. **C** $\quad -x(x - 3)$ $\qquad -x^2 + 3x = C$

10. **N** $\quad 2x(x + 4)$ $\qquad 2x^2 + 8x = N$

11. **T** $\quad -3x(x - 2)$ $\qquad -3x^2 + 6x = T$

12. **M** $\quad -2x(3x - 5)$ $\qquad -6x^2 + 10x = M$

13. **K** $\quad 2(x + y) + (x - y)$ $\quad 3x + y = K$

14. **I** $\quad 3(x + y) + (x + 2y)$ $\qquad 4x + 5y = I$

15. **Y** $\quad 4(2x - 3) - 2(x + 4)$ $\qquad 6x - 20 = Y$

16. **G** $\quad 3(3x + 2) - 2(x - 4)$ $\qquad 7x + 14 = G$

A	P	E	R	S	O	N
$3x + 3y$	$6x - 6y$	$-2x - 2y$	$-3x + 3y$	$3x - 9$	$-2x - 8$	$2x^2 + 8x$

W	H	O		C	A	N	'	T
$3x + x^2$	$x^2 - 2x$	$-2x - 8$		$-x^2 + 3x$	$3x + 3y$	$2x^2 + 8x$		$-3x^2 + 6x$

M	A	K	E		A
$-6x^2 + 10x$	$3x + 3y$	$3x + y$	$-2x - 2y$		$3x + 3y$

M	I	S	T	A	K	E
$-6x^2 + 10x$	$4x + 5y$	$3x - 9$	$-3x^2 + 6x$	$3x + 3y$	$3x + y$	$-2x - 2y$

C	A	N	'	T		M	A	K	E
$-x^2 + 3x$	$3x + 3y$	$2x^2 + 8x$	$-3x^2 + 6x$			$-6x^2 + 10x$	$3x + 3y$	$3x + y$	$-2x - 2y$

A	N	Y	T	H	I	N	G
$3x + 3y$	$2x^2 + 8x$	$6x - 20$	$-3x^2 + 6x$	$x^2 - 2x$	$4x + 5y$	$2x^2 + 8x$	$7x + 14$

QUICK QUESTIONS

1. Find the value of C when F = 67 and C = $\frac{2}{5}$(F - 32) $C = 14$

2. Simplify -2a + 5b + 5a - 7b $3a - 2b$

3. Simplify 5a × 2b × c $10abc$

4. Simplify $5x^2$ - 5x - 6 + 8x $5x^2 + 3x - 6$

5. Write a rule for the pattern 3, 6, 9, 12, ... $3n$

6. Write a rule for the pattern 2, 5, 8, 11, ... $3n - 1$

7. Simplify $x^7 × x^3 × x^2$ x^{12}

8. Simplify $4x^2 × 6x^3$ $24x^5$

9. Simplify $\frac{x^{10}}{x^3}$ x^7

10. Simplify $\frac{8x^{10}}{4x^3}$ $2x^7$

PUZZLE

1. Start with a number.

2. Add 2. $+ 2 =$
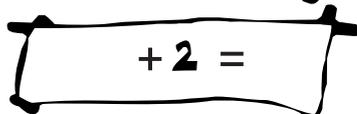
3. Multiply this by 3. $×3 =$
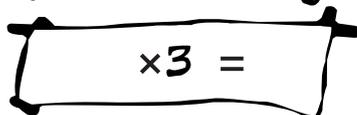
4. Subtract 6 from the total. $- 6 =$
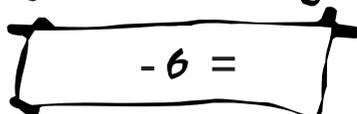
5. Divide by 3. $÷ 3 =$
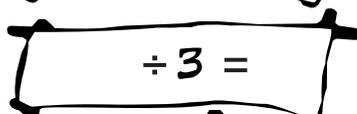
6. What is the number that was left?
   Explain why

BLACK Worksheet 11

Factorials are useful in working out the number of ways things can be selected or arranged. For example, 3 people can be arranged in a line 3! or 6 different ways.
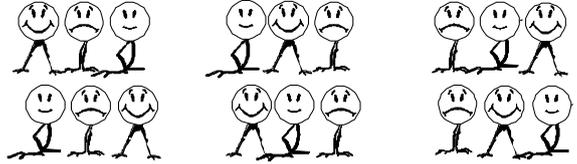
1 Write the first seven factorials:

$0! = \underline{\quad 1 \quad}$

$1! = \underline{\quad 1 \quad}$

$2! = \underline{\quad 2 \quad}$

$3! = \underline{\quad 6 \quad}$

$4! = \underline{\quad 24 \quad}$

$5! = \underline{\quad 120 \quad}$

$6! = \underline{\quad 720 \quad}$

2 Explain why 6 × 5! gives the same answer as 6!

$6! = 6 \times 5 \times 4 \times 3 \times 2 \times 1$

$5! = 5 \times 4 \times 3 \times 2 \times 1$

Therefore $6! = 6 \times 5!$

3 If 8! = 40 320 calculate 9!

$40\ 320 \times 9 = 362\ 880$

4 $(0! + 0! + 0! + 0! + 0!)! = \underline{\quad 120 \quad}$

5 Simplify $\dfrac{7!}{5!} = \dfrac{7 \times 6 \times 5!}{5!}$

$= \underline{\quad 42 \quad}$

BEDMAS

1. $4 + 3 \times 2 = \underline{\quad 10 \quad}$

2. $5 + 7 \times 8 = \underline{\quad 61 \quad}$

3. $9 \times 9 - 5 \times 8 = \underline{\quad 41 \quad}$

4. $36 \div 4 + 15 = \underline{\quad 24 \quad}$

5. $4 \times (8 + 7) = \underline{\quad 60 \quad}$

6. $(2 \times 5^2) - 4 \times 5 = \underline{\quad 30 \quad}$

7. $(8 + 7) \times (12 - 9) = \underline{\quad 45 \quad}$

8. $(7 + 4)^2 - 6 \times 8 = \underline{\quad 73 \quad}$

9. $3^2 \times 7 - 5 \times 5 = \underline{\quad 38 \quad}$

10. $3! \times (15 - 9) \div 2 = \underline{\quad 18 \quad}$

11. $[(3! + 2)^2 - 40] \times 2 = \underline{\quad 48 \quad}$

12. $3! \times (15 - 9) \div 3 = \underline{\quad 12 \quad}$

THE RESTLESS SEA

A tsunami is an ocean wave that occurs after an earthquake (or some other eruption or explosion) takes place in or near the sea. The wave moves very fast in deep water and can have speeds of over 900 km/hour. The wave slows as it reaches shallow water and the energy of the wave's speed is then transformed to increased height and force. As it slows it rises to a great height and can cause enormous destruction.

An undersea earthquake in the Indian Ocean on 26th December 2004 produced a tsunami that killed over 200,000 people. The earthquake took place at about 8am (local time) in the Indian Ocean off the western coast of northern Sumatra and the resultant tsunami hit the coast of Somalia (4500 km away) around 7 hours later.

1. What was the speed (in metres per second)?

    7 hours = 25200 seconds

    4500 km = 4500 000 m

    4500 000 m ÷ 25 200 seconds

    = 179 metres / sec

2. Convert this figure to kilometres per hour.

    179 m/sec × 60 (sec) × 60 (min) ÷ 1000 (metres)

    = 644 km / hour

3. The speed of the tsunami (in metres per second) is related to the depth of the water by the equation $s^2 = 9.81d$ where s = speed and d = depth of the water. Suppose an earthquake at sea produces a tsunami in water 700 metres deep. Determine the speed of the tsunami wave traveling through this area.

    $(speed)^2 = 9.81 \times 700$

    $(speed)^2 = 6867$

    speed = 82.87 metres per second.

4. Fill in the missing word:
    "As ocean depth increases the speed of the tsunami   _increases_   "

5. Why would determining a tsunami's speed be a useful skill?

    Scientists (or mathematicians) can calculate the speed in relation

    to the ocean depth and then send information to tsunami

    warning centres.

METHODS OF MULTIPLICATION

The Lattice Method Of Multiplication

1. Draw a rectangle and divide it into squares.
   Draw a diagonal in each square. Write the first number at the top.
   Write the second number on the right hand side.

2. Multiply each digit. Write the product in each half of the square.

3. Add the numbers diagonally.

Look at these examples:

Step 1   *Step 2*   *Step 3*

642 × 8

= 5136

69 × 53

= 3657

Use the Lattice Method of Multiplication for each of the following:

1. 583 × 7

   = 4081

2. 678 × 5

   = 3390

3. 46 × 82

   = 3772

4. 27 × 42

   = 1134

5. 58 × 39

   = 2262

6. 967 × 25

   = 24,175

MATHS IN ACTION

1. A company has an advertising budget of $25,000. It plans to produce and air a radio commercial to promote its products. It will cost $5000 to produce the commercial and an additional $250 each time the commercial is aired on the radio breakfast show. How many days can the company afford to run the commercial if it is aired once a day?

$25000 – $5000 = $20 000

$20 000 ÷ $250 per day = 80 days

2. A bread-making machine costs $599. The ingredients to make a 1 kg loaf of bread are milk powder $4.50 (1 packet makes 15 loaves), yeast $5 (makes 5 loaves), flour $4.50 (makes 6 loaves), packet of salt $3 (contains enough for 150 loaves). At a supermarket you pay $4.50 for the same size loaf of bread. How many whole loaves of bread will you have to make in order for the cost of the machine and ingredients to be less than the cost of buying an equivalent amount of bread at the store?

Cost per loaf = ($4.50 ÷ 15) + ($5 ÷ 5) + ($4.50 ÷ 6) + ($3 ÷ 150)

= $2.07 per loaf

Equation $599 + 2.07x = 4.5x

599 = 2.43x

x = 246.5 (247 loaves will have to be baked)

Note this does not take into account electricity to run the machine.

3. Patrick has 50 millilitres of a 30% solution of copper sulfate and a further bottle of 20% solution. For his chemistry experiment, he needs a 22% solution of copper solution. He makes the following table to calculate the amount that he needs to mix to get a 22% solution.

	Amount of Solution (mL)	Amount of Copper Sulphate
30% solution	50	0.30(50)
20% solution	x	0.20x
22% solution	50 + x	0.22(50 + x)

He then forms an equation to solve: 0.30(50) + 0.20x = 0.22(50 + x)
Solve the equation and find how much of the 20% solution should be added.

15 + 0.2x = 11 + 0.22x

4   = 0.02x

x   = 200 mL

200 mL of the 20% solution should be added.

BLACK Worksheet 12

Complete the missing numbers from the cross number.

$$60 \div 10 = 6$$

$$\div \quad \times \quad \times$$

$$15 \div 5 = 3$$

$$= \quad = \quad =$$

$$200 \div 4 = 50 \qquad 18 \div 3 = 6$$

$$72 \div 2^3 = \sqrt{81}$$

$$\div \quad \times \quad \div$$

$$63 \div 7 = 9 \qquad 3! \div 3 = 2$$

$$\times \quad = \quad =$$

$$2^3 \qquad 48 - 3 = 45$$

$$\times \quad = \quad \div$$

$$48 \div 8 = 3! \qquad 3^2 \qquad 42 \div \sqrt{36} = 7 \qquad 3^2$$

$$\div \quad \times \quad = \quad =$$

$$4 \times 7 = 28 \qquad 27 + \sqrt{25} = 32 \qquad 2^3 - 3 = 5$$

$$= \quad = \quad \div \quad \div$$

$$12 \qquad 56 - 4 = 52 \qquad 96 \div 2^3 = 12 \qquad 3$$

$$\div \qquad \div \quad = \quad - \qquad =$$

$$92 \div 4 = 23 \qquad 6 + 4 = 10 \qquad 1$$

$$4! \qquad = \qquad = \qquad =$$

$$\div \qquad 13 + \sqrt{9} = 16 \qquad 5! \div 2 = 60$$

$$37 + 6 = 43 \qquad \div \qquad \div \qquad \div$$

$$= \quad - \qquad 5! \div \sqrt{16} = 30 \qquad 3! + \sqrt{25} = 11$$

$$2^2 + 18 = 22 \qquad = \qquad = \qquad =$$

$$= \qquad 4 \qquad 4 \times 3 = 12$$

$$25 - 4! = 1$$

LINEAR PATTERNS
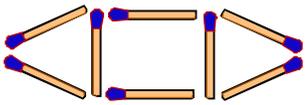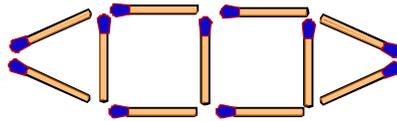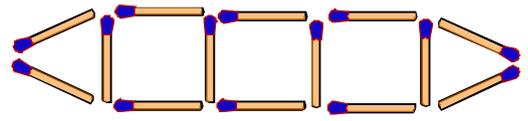
Pattern 1   Pattern 2   Pattern 3

1. Complete the table below to show the number of match sticks used for each pattern.

Pattern number (n)	1	2	3	4	5	6
Matchsticks (m)	8	11	14	17	20	23

2. Julia says "Because you add three to get the next number in the sequence, the rule should be 3 times the pattern number, then add 5."

Write Julia's rule as an equation. $3n + 5$

Another matchstick pattern is drawn below.
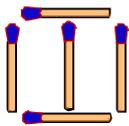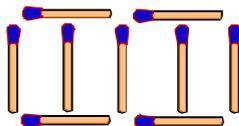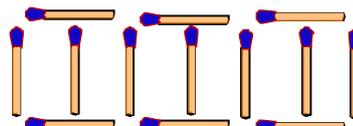
Pattern 1   Pattern 2   Pattern 3

3. Draw a diagram of pattern number 4 in the space below.
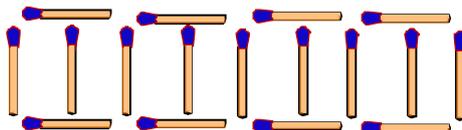
4. Complete the table to show the number of matchsticks needed for pattern numbers 3, 4 and 5.

Pattern number (n)	1	2	3	4	5	6
Matchsticks (m)	5	9	13	17	21	25

5. Which pattern number needs exactly 41 matchsticks? _____ *pattern 10*

6. How many matchsticks are needed for pattern 50?   *201 matchsticks*

7. Julia says "Because you add four to get the next number in the sequence, the rule should be 4 times the pattern number, then add 1."

Write Julia's rule as an equation: $4n + 1$

WRITING AN EQUATION FROM A TABLE

Bamboo is one of the fastest growing plants on earth. In some countries it can grow over 1 metre per day. The table below shows a bamboo plant's observed growth over an 8 hour period.

1. Graph the observations and show that the table represents a linear function (i.e. draws a straight line graph).

Time (hours), x	0	2	4	6	8
Height (mm), y	150	250	350	450	550

Bamboo Growth
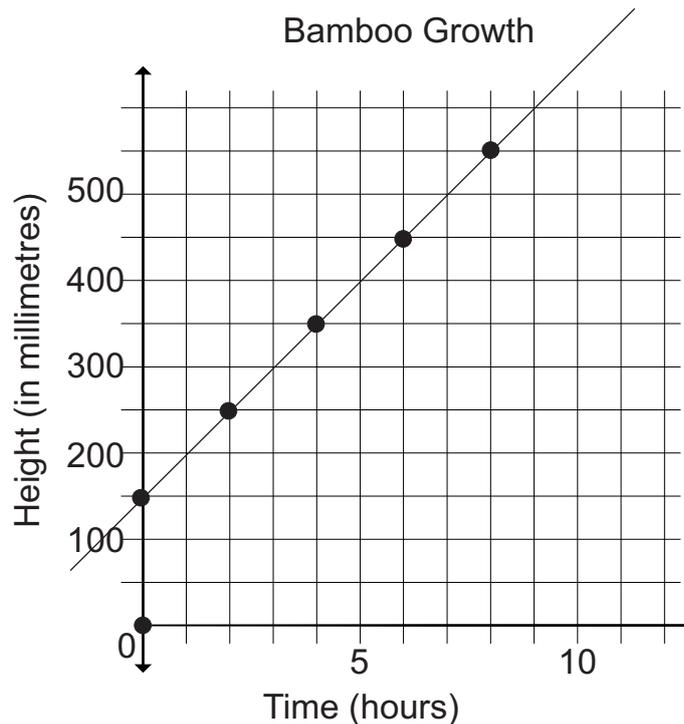
2. Write an equation for the function so that the height of the bamboo could be calculated given any time in hours.

   The equation starts at x value = 0, y value = 150

   Gradient of the graph is 50

   (for every 1 hour the gradient increases by 50 mm)

   Therefore the equation is y = 50x + 150

   Where y = height of the bamboo plant in mm

   x = time in hours.

3. Use your equation to calculate how many hours it would take before this particular bamboo plant grows 1 metre tall (assuming that growth continues at the same rate).

   Put the values into the equation (1000mm = 1m), 1000 = 150 + 50x

   Solving the equation you will find that the time taken = 17 hours

GRAPHING AN IRRATIONAL NUMBER

D

1 unit

A   B   C

1 unit

Point = 1.4
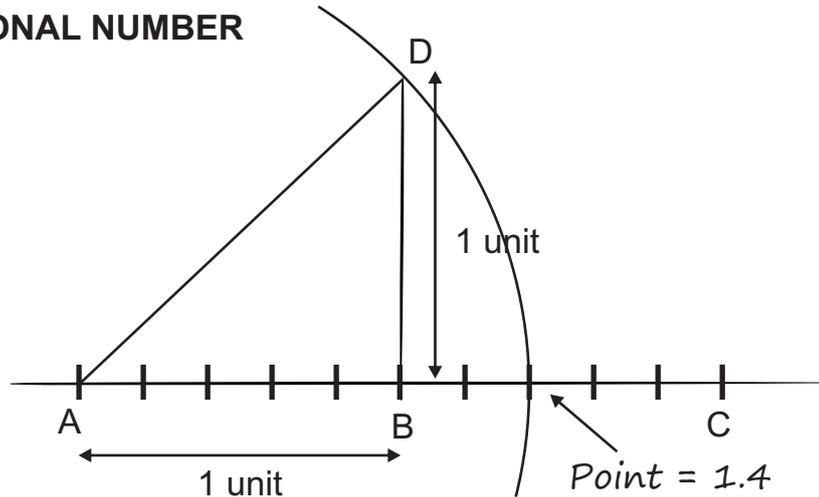
1. The diagram above shows a triangle with two sides each with a length of 1 unit. It has an arc with radius AD. Line AC is divided into 10 equal parts.
Prove that the triangle is a right angled triangle.

Using Pythagorus $AD^2 = AB^2 + BD^2$

This means $AD = \sqrt{2}$ or 1.41 units

If AB = 1 unit, then each of the equal parts is 0.2

The arc crosses at 1.4, close to the hypotenuse of length 1.41 units

2. What is an irrational number?

A number that cannot be expressed as a fraction

Indicate on the number line below the approximate position of each of the following: $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, $\sqrt{6}$, $\sqrt{7}$, $\sqrt{8}$, $\sqrt{10}$

All the dots are in numerical order

0   1   2   3   4
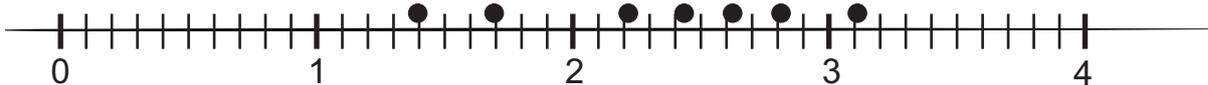
You are going to draw the "Wheel of Theodorus as shown below. Start near the centre of a large piece of paper. Draw a right angled triangle with sides 1 unit in length. Next, using the hypotenuse as the base, draw another right angled triangle with the outside leg as 1 unit. Repeat this process to draw at least 20 right angled triangles. Colour your "Wheel of Theodorus". What are the lengths of each hypothenuse? Use mathematics to calculate the first 3 hypotenuse lengths
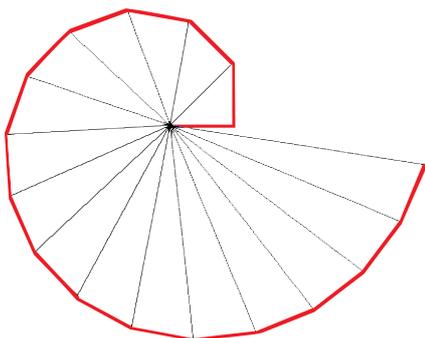
The first triangle $\sqrt{1^2 + 1^2} = \sqrt{2}$

2nd triangle $\sqrt{\sqrt{2}^2 + 1^2} = \sqrt{3}$

3rd triangle $\sqrt{\sqrt{3}^2 + 1^2} = \sqrt{4}$

A pattern emerges. The rest of the hypotenuse

will be the square roots of 5, 6, 7, 8, 9 .....

BLACK Worksheet 13

EXPANDING AND FACTORISING

Expand

$4(x + 7)$ → $4x + 28$
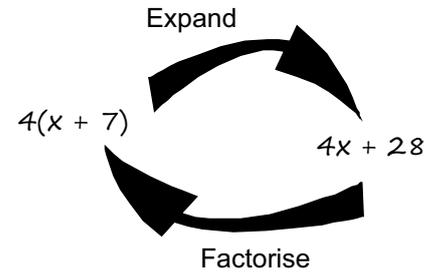
Factorise

1. Why do we need to factorise?
    Why do we need to expand?

    We factorise or expand to help give us more options when
    we are trying to simplify, or solve an equation.

2. Complete this table of expressions in factorised and expanded form.

Factorised Form		Expanded Form
$3(x + 9)$	↔	$3x + 27$
$5(2x + 3)$	↔	$10x + 15$
$x(7 - y)$	↔	$7x - xy$
$2a(b - 8)$	↔	$2ab - 16a$

    ← 7 metres →

    Area = $7x$ m^2 — x metres

    Area = 49 m^2 — 7 metres

    $7(x + 7) = 7x + 49$
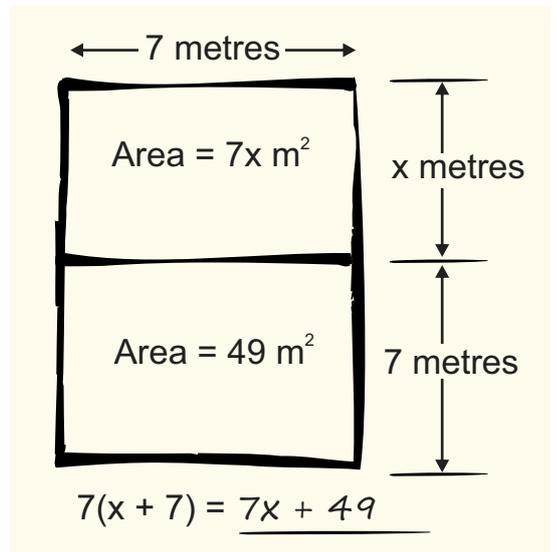
3. Expand the expressions below

    $3(x + 7)$ = $3x + 21$

    $4(x + 5)$ = $4x + 20$

    $x(2x + 7)$ = $2x^2 + 7x$

    $2x(3x + 9y) = 6x^2 + 18xy$

    $4(x - 2)$ = $4x - 8$

    $3(x - 8)$ = $3x - 24$

    $x(3x - 17)$ = $3x^2 - 17x$

    $3x(3x - y)$ = $9x^2 - 3xy$

    $3(2x + 2)$ = $6x + 6$

    $2(5x + 6)$ = $10x + 12$

    $4(3x - 2)$ = $12x - 8$

    $7(4x - 3)$ = $28x - 21$

    $-4(1 + 2x)$ = $-4 - 8x$

    $-1 (3 - x)$ = $-3 + x$

    $2x(x + 4)$ = $2x^2 + 8x$

    $-x(1 + x)$ = $-x - x^2$

TRIANGULAR NUMBERS

The diagram below shows how a tower can be made out of snowballs.
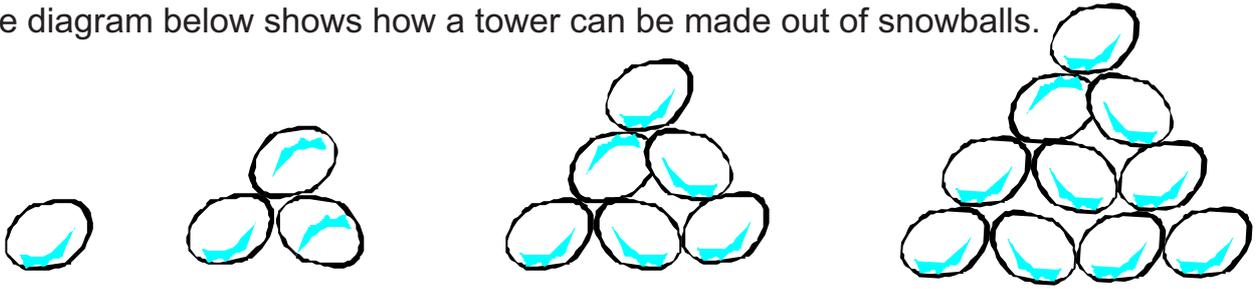
1. Draw the next tower in the pattern (above), then complete the table below.

Tower Number (n)	1	2	3	4	5	6
Snowballs (s)	1	3	6	10	15	21

2. The number of balls needed to build each tower forms a sequence called triangular numbers.

The formula for calculating the nth term of a triangular number is: $\dfrac{n^2 + n}{2}$

Use the formula to calculate how many snowballs would be needed for Tower 10.

55 snowballs

3. If you had 120 snow balls, what would be the Tower Number?

Tower 15

4. Here are the first five terms of a number sequence: 5, 8, 11, 14, 17.

Circle the correct expression for the nth term of the sequence.

$5n$ $n^2 + 4$ $n + 4$ $3n + 2$

5. Look at the three sequences of numbers below:
Sequence p: 4, 6, 8, 10, 12, . . .
Sequence q: 3, 8, 15, 24, 35, . . .
Sequence r: 5, 10, 17, 26, 37, . . .

Sequence r is obtained from p and q by the following formula: $r = \sqrt{p^2 + q^2}$

Calculate the sixth term of each of the sequences p, q and r.

p = 14 (add 2), q = 48 (add 13)

$r = \sqrt{14^2 + 48^2}$

$r = $ *50*

PERCENTAGES

1. Elizabeth sees an advertisement for a new stereo. She can purchase the stereo on "time payment" with a deposit of 25% and then 18 monthly installments of $45.Complete the calculations in the table below to calculate how much extra Elizabeth will eventually pay by buying the stereo on time payment.

NEW STEREO $800
Ask about our hire purchase terms!

25% Deposit	$800 × 0.25 = $200
Total Cost of 18 monthly payments @ $45	$810
Total cost Elizabeth will pay	$1010
Advertised Cost of the Stereo	$800
Extra amount Elizabeth has to pay	$210

2. Elizabeth is looking to purchase a new shirt and jeans. She could either buy them ready made or decide to save money and make them herself.
.   The cost of a new shirt and jeans is below. Find the total cost.

Shirt $59.95, Jeans $99.99, Total Cost $ _____ $159.94 _____

Elizabeth decides to save money by making the shirt herself. She calculates that she will need two metres of material. This material usually costs $18.45 per metre, however at sale price she can save 10%. Elizabeth also need to purchase a shirt pattern ($15.95), 6 buttons (1.55 cents each) and a reel of cotton ($3.30). How much does Elizabeth save by making her own shirt?

2 metres of material @ $18.45	=	$36.90
10 % discount	=	$3.69
Sale Price	=	$33.21
Shirt Pattern @ $15.95	=	$15.95
6 buttons @ $1.55	=	$9.30
Cotton @ $3.30	=	$3.30
Total Cost	=	$61.76
Saving	=	–$1.81

Comment on the savings.

It is cheaper to buy the shirt, however subsequent shirts that she makes will be cheaper now that she has the pattern and cotton.

CORN CALCULATIONS

Two farmers each harvest 50 hectares of corn per day from their fields. The area of one farmer's field is 1000 hectares, and the area of the other farmer's field is 600 hectares.

1. The equation y = 1000 - 50x gives the unharvested area (y) of the larger field (in hectares) after x days. Write an equation giving the unharvested area y of the smaller field (in hectares) after x days.

y = 600 - 50x

2. Graph the two equations from 1. on the grid below.

Corn Harvesting
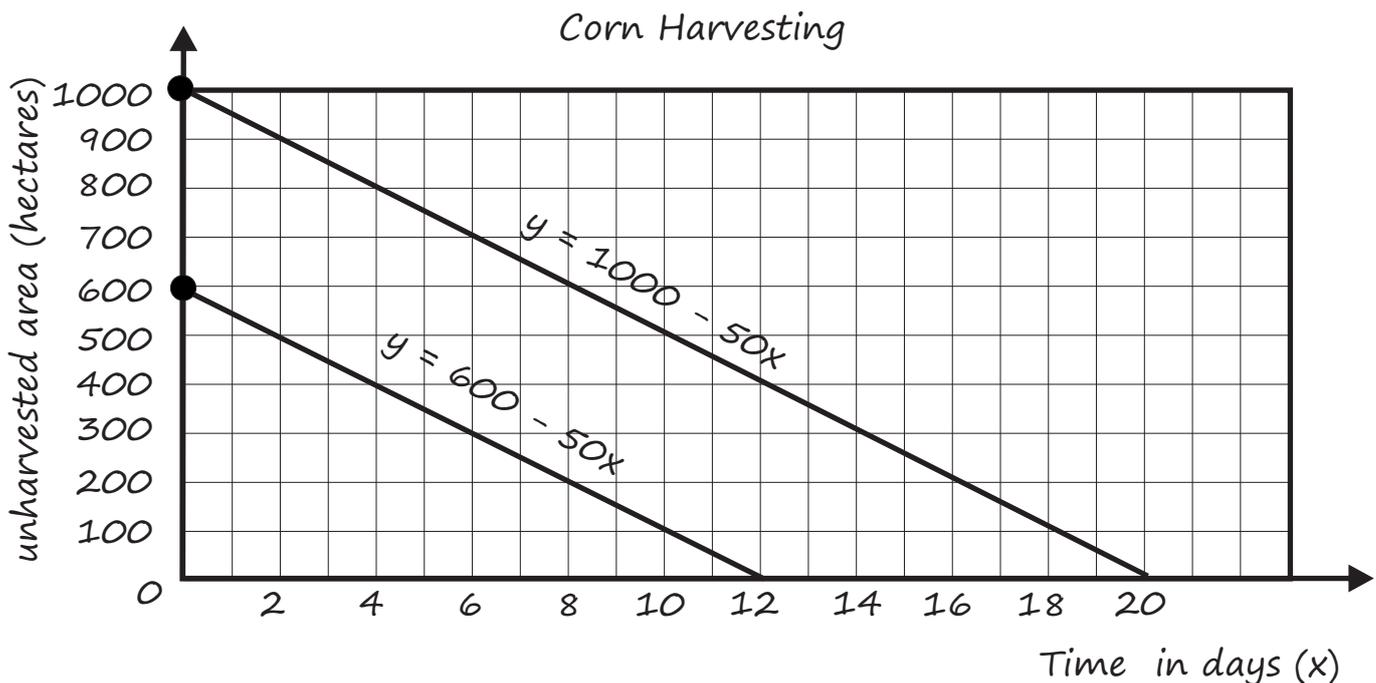
3. Identify the slope and y-intercept of each graph.

For y = 1000 - 50x, the slope is -50 and the y-intercept is 1000.

For y = 600 - 50x, the slope is -50 and the y-intercept is 600.

4. What is the geometric relationship between the two line graphs? How do you know?

Both lines are parallel. You know this as they have the same slope (of -50) but different y-intercepts.

5. How long does it take for the farmers to harvest the corn in both fields?

The large field 20 days

The smaller field 12 days

BLACK Worksheet 14

QUICK QUESTIONS

1. $3 + 6 \times 4 + 5$ *32*

2. $15 \div 3 \times 6 - 4^2$ *14*

3. $2(10) + 3(4 + 8)$ *56*

4. $2[\,5 + (45 \div 9)^2\,]$ *60*

5. $\dfrac{5 + 4^2}{3^2 \times 4}$ *0.583*
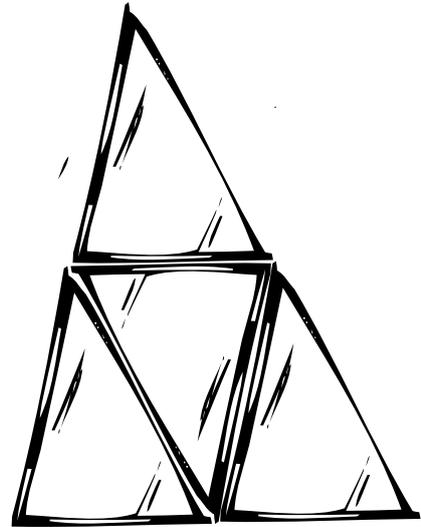
The above large triangle is made of identical smaller triangles each with a base of 2.6 cm and vertical height of 3.4 cm.

Calculate the area of the large triangle.

17.68 cm²

6. If a = 7, b = 3 and c = 5 then evaluate $a^2 - (b^3 - 5c)$.

47

7. If m = 10, n = 5, p = 2 then evaluate $m^3 + n + p^3$.

1012

8. If r = 9, s = 12, t = 15 then evaluate $3rs - t^2$.

99

9. If g = 4, h = 6, j = 8 and k = 12 then evaluate $2k^2 + \dfrac{2g(h - g)}{gh - j}$

289

10. If x = 12, y = 4, z = 3 then evaluate $\dfrac{xy^2 - 3z}{3}$

61

TAKING A BREAK

Edith and John travel to Hawaii for 5 day's holiday. Airfares cost $1200 + GST each (GST is charged at 15%). Together, their accommodation costs are $1500. Airport tax is $25 each. Travel insurance is $60 each. They have also budgeted for $300 per day (each) for meals and other purchases (per diem).

1. What is the total cost?

Airfares	$2400
GST	$ 360
Accommodation	$1500
Airport Tax	$ 50
Travel Insurance	$ 120
Per Diem	$3000
Total	$7430

2. The travel agency arranged a 20% discount on their travel insurance.
If they each paid $60, how much was the cost of their original policies?

$60 ÷ 0.8 = $75 (original price)

3. While in Hawaii, Edith and John hire a car to travel around the island.
Hire charges are $40 per day with an extra charge of 16 cents per kilometre.
They keep the car for 4 days and travel a total of 935 km.
How much does it cost them for the car hire?

$40 × 4 + 0.16 × 935 = $309.60

4. During one drive to a surf beach, they travel at an average speed of 90 km/h for 3 hours. On the drive back they average 60 km/hr.

How far is it to the surf beach?   90 km/h × 3 h = 270 km

How long did it take them to drive back? 270 km ÷ 60 km/h = 4.5 hours

5. The car uses 40 litres of petrol for the trip to the beach and back. Petrol costs $1.70 cents per litre. Calculate the average fuel consumption per 100 kilometre and calculate much John and Edith would have paid for petrol.

Total distance (to beach and back) = 540 km

40 litres ÷ 5.4 = $7.40 litres per 100 km

40 litres × $1.70 = $68

6. For their flight home, the plane is due to leave for New Zealand at 22:15. It is delayed for 2½ hours. What time will they leave?

0045 i.e. 12.45 am, or quarter to 1 in the morning

WILL YOU SPONSOR ME?

Delmaine participates in a walk-a-thon. Donors can pledge money for each kilometre that she walks and / or a fixed amount that doesn't depend on how far she walks. The table gives the amounts pledged by four donors.

	Donor	Amount per km (x)	Fixed amount	Equation
(1)	Praveen	None	$50	$y = 50$
(2)	Sanjit	$2	$20	$y = 2x + 20$
(3)	Salil	$5	None	$y = 5x$
(4)	Peyton	$3	$40	$y = 3x + 40$

1. For each person, write an equation giving the amount of money **(y)** that person will donate if Delmaine walks x kilometres.

2. Graph your equations on the grid below.

Walk-A-Thon
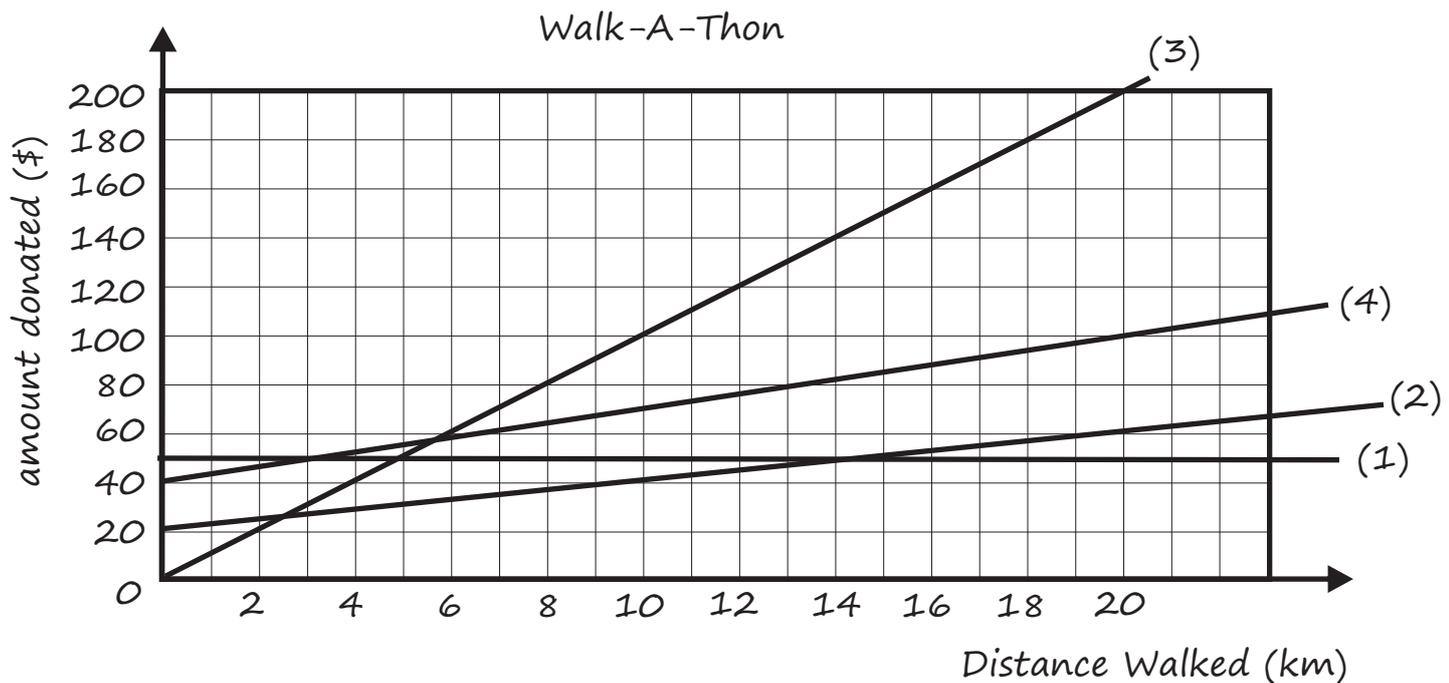
3. Consider the equations you have written. Which donor will potentially give the most money? Explain why this is so.

   Potentially Salil will donate the most as the gradient of his line is the greatest. It really depends on how far Delmaine walks. For the first 3km Praveen is giving the most, then Peyton has pledged the most. After 6 km it is Salil who will be donating the most.

MATHS CHALLENGE

The first four triangles in a pattern are drawn below.
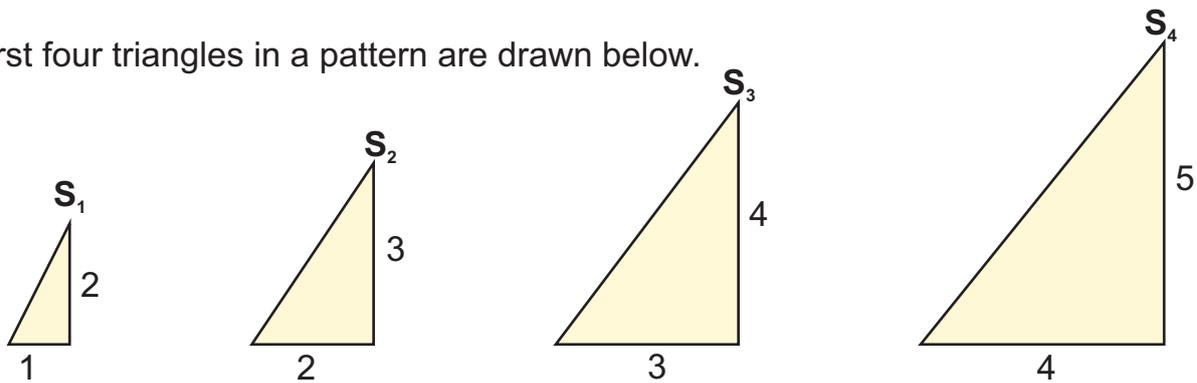
What would be the area size of the nth triangle (S_n)

base length = n	
height length = n + 1	
Area = ½bh	
Area = ½n(n + 1)	
Area = ½n² + ½n	

MATHEMATICAL PROBLEM SOLVING

Since 1912, scientists have been mapping the glaciers of Mount Kilimanjaro in Africa. Their data suggests that the glaciers are shrinking in overall area.
Below is a table of their results to date:

Year	1912	1953	1976	1989	2000	2006
Glacier Area (km²)	12.1	6.7	4.2	3.3	2.2	2.0

Let x be the number of years since 1912 and let y be the area of the glaciers (in square kilometres). Make a scatter plot of the data pairs and draw a line of best fit through the data points. Predict the year when the glaciers will disappear.
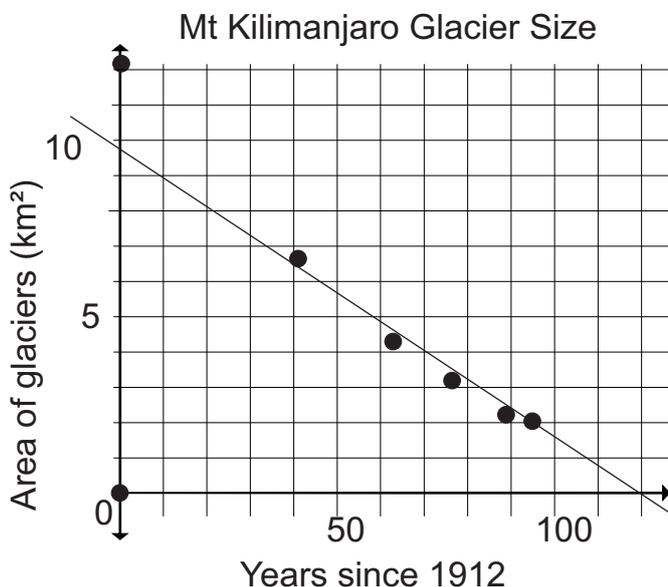
By ignoring the 1912 value a line of best fit seems to indicate that by 2020 the glaciers will disappear. This is what scientists also generally agree on. However in 2006, an El Nino weather pattern increased the glacier peaks. More recent data might push back the final date.

BLACK Worksheet 15

EQUATION SOLVING

1. $8x - 1 = 10x - 2$

$-2x = -1$

$x = 0.5$

2. $7x + 10 = 8x - 32$

$-x = -42$

$x = 42$

3. $4x + 14 = 10(x - 7) - 3x$

$14 = 10x - 70 - 3x - 4x$

$84 = 3x$

$x = 28$

4. $2(x - 3) = 3(x - 1)$

$2x - 6 = 3x - 3$

$-x = 3$

$x = -3$

5. $5x = 2x - 18$

$3x = -18$

$x = -6$

A BAFFLING BIRTHDAY

Cheyenne is having an end of year party. There is something baffling about the date of her birthday in that two days before the party she was 15 years old but at the same time next year she will be 17 years old. How can this be true?

Cheyenne must be at a New Year's party. Two days before it was December 30 (she was 15 years old) The next day was her birthday (Dec 31, 16 years old). In one years time she will be a year older (17 years old).

A CORD OF WOOD

A cord is a unit of measure of dry wood. It is defined as 128 cubic feet (or 3.62 m^3). If you were designing a trailer to fit a cord of wood, what would be a possible dimension of the trailer?

e.g 1.5m × 1.5m × 1.6m

Let $n = 3$, $a = 5000$ and $b = -\dfrac{1}{25}$

Write the value of $(ab)^n$ in words.

Let $b = -0.04$

$(5000 \times -0.04)^3 = 8\ 000\ 000$

$= 8$ million

Calculate $\dfrac{1500 \times 15}{25 \times 60}$

$1500 \times 15 = 22\ 500$

$25 \times 60 = 1500$

$22\ 500 \div 1500 = 15$

DEALING WITH DAIRY DATA

The table below shows Dairy Production for the last 10 years at one central North Island factory.

Year Ended 31 March	CENTRAL DAIRY PRODUCTION Dairy Products (thousand tonnes)			
	Butter	Cheese	Powdered Milk	Casein
2000	247	102	197	45
2001	240	101	188	48
2002	241	88	192	50
2003	244	88	264	45
2004	265	109	206	47
2005	277	85	203	82
2006	243	78	174	81
2007	252	92	176	80
2008	261	107	176	85
2009	266	88	188	87
2010	251	115	119	87

1. In which year did Central Dairy Production produce the most Butter?

Year = *2005*

2. Write Casein as a fraction and a percentage of total Dairy Production for 2007.

Fraction = $\frac{80}{600} = \frac{2}{15}$

Percentage = *13.3% (1 DP)*

3. Which product had the biggest increase in production over the ten years?

Casein production has nearly doubled since 2000

Product = *Casein*

4. Calculate the total amount (in tonnes) of Dairy Products that were produced in the year ending March 31, 2010. Give your answer to two significant figures.

Total = *570,000 2 sf*

5. Percentage increase is defined as the increased amount divided by the original amount. Use this information to give the percentage increase or decrease of cheese and of powdered milk in the 11 years from 2000 to 2010.

		Cheese	Powdered Milk
2000 figure	*+ = an increase*	*102,000*	*197,000*
2010 figure	*– = a decrease*	*115,000*	*119,000*
Increase or decrease amount		*13,000*	*–78,000*
Percentage increase or decrease		*13000÷102000*	*–78000÷197000*
		= 12.7%	*= –39.6%*

CLIMATE CALCULATIONS

1. In Auckland, 1445 mm of rain fell during 2010.

   Write 1445 mm in metres. _____ = 1.445 m_____

2. During 2010, there was four times more rain in winter than the rest of the year.

   Calculate how much rain fell in the winter. $\frac{4}{5}$ × 1445 = 1156 mm

3. Rainfall is measured in a rain gauge. One type of rain gauge is shown below.
   Calculate the total volume of water that the gauge could hold.
   HINT: (Volume = Cross Sectional Area × Length)

   Area triangle = ½ × 100 × 400

       Volume = ½ × 100 × 400 × 100

       Volume = 2,000,000 mm³

   100 mm
   100 mm
   400 mm
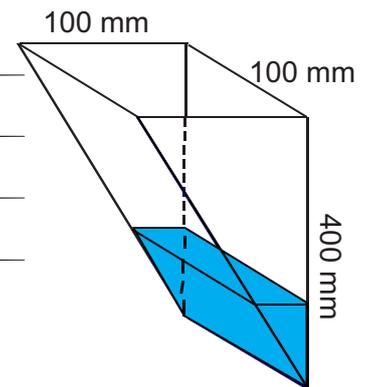
4. If full, how many litres of water would the gauge hold?

       2 litres

This table gives the amount of rainfall, for two years in Auckland, during Aug and Sep.

	2009	2010
August	180 mm	131 mm
September	195 mm	156 mm

5. What is the total amount of rainfall for August and September 2010?

       Total = 287 mm

6. The rainfall in October 2010 was 121 mm. This was 10% more than October 2009. Calculate the amount of rainfall in October 2009

       121 ÷ 1.1 = 110 mm

7. Edith draws a graph (below) based on the results in the table. In her project, Edith states "There was a 49% difference in the amount of rainfall for August 2009 & August 2010". Explain why Edith is wrong.

August Rainfall

200
150
100
50
0
   2009   2010
Year
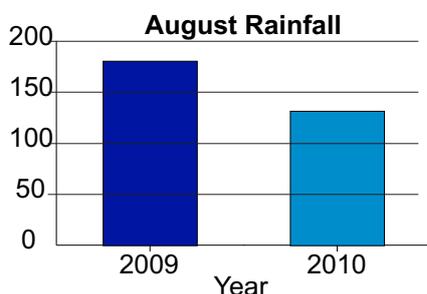
Percentage Difference is 49 ÷ 180 = 0.27

There was a 27% difference. Edith has

confused the actual difference with the

percentage difference.

Factorise the following

$x^2 + 10x + 24$ *(x + 6)(x + 4)*	$x^2 + 9x + 8$ *(x + 8)(x + 1)*	$x^2 + 13x + 30$ *(x + 10)(x + 3)*
$x^2 + 2x - 48$ *(x + 8)(x − 6)*	$x^2 + 6x - 40$ *(x + 10)(x − 4)*	$x^2 + 4x - 5$ *(x + 5)(x − 1)*
$x^2 - 8x + 15$ *(x − 5)(x − 3)*	$x^2 - 9x + 14$ *(x − 7)(x − 2)*	$x^2 - 22x + 120$ *(x − 10)(x − 12)*
$x^2 - 2x - 35$ *(x − 7)(x + 5)*	$x^2 - x - 72$ *(x − 9)(x + 8)*	$x^2 - 11x - 12$ *(x − 12)(x + 1)*
$x^2 - 10x + 25$ *(x − 5)(x − 5)*	$x^2 + 20x + 100$ *(x + 10)(x + 10)*	$x^2 - 100$ *(x + 10)(x − 10)*

Factorise the following expression

$x^2 - y^2$ *(x + y)(x − y)*

Use your answer above, to work out the exact value of: $123456789^2 - 123456788^2$

(123456789 + 123456788) = 246913577

(123456789 − 123456788) = 1   therefore answer = 246913577
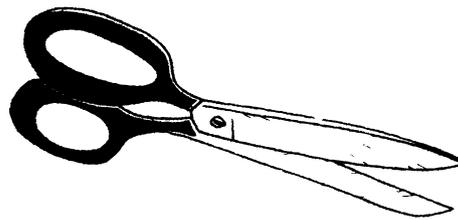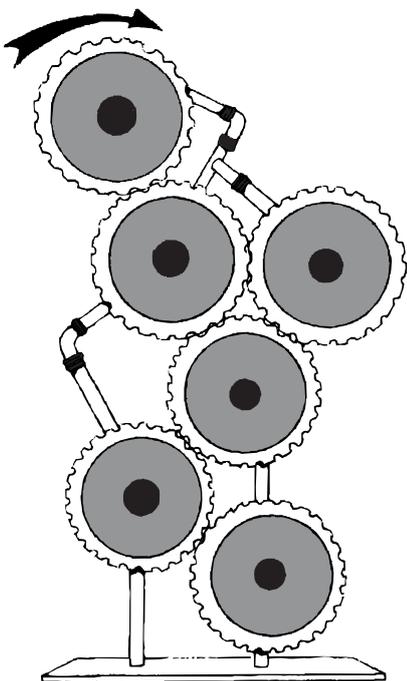
Some non-mathematicians have come up with two inventions - the Gear Wheel Machine and some new scissors. However each has a problem.

What is the problem with the Gear Wheel Machine?
The three middle gears joined like a triangle would lock and the wheels would not to rotate.

What is the problem with the scissors?
They don't close properly

BLACK Worksheet 16

EQUATION SOLVING

1. $6x + 7 = 8x - 13$

$-2x = -20$

$x = 10$

2. $5(x - 2) = 30$

$5x - 10 = 30$

$5x = 40, x = 8$

3. $8x - 3 = 5(2x + 2)$

$8x = 10x + 10 + 3$

$-2x = 13$

$x = -6.5$

4. $2(3x + 1) + 2x = 12x$

$6x + 2 + 2x = 12x$

$2 = 4x$

$x = 0.5$

5. $4(x - 2) = 8x + 2$

$4x - 8 = 8x + 2$

$-4x = 10$

$x = -2.5$

Not drawn to scale

$5°$

PERCENTAGE DISTRIBUTION

Leaving a "tip" originated in 18th century English coffee houses. It meant "To Insure Promptness". In some countries a tip of 15% is expected when you eat at a restaurant or have room service in a hotel.

Five parties eat at a restaurant. The total amount (without tip) for each account is listed below:

Party 1: $195.50
Party 2: $240.20
Party 3: $180.80
Party 4: $255.60
Party 5: $160.40

The waitress expects to make a 20% tip on each of the totals. To calculate the tip should she multiply each amount by 0.2 and then add to find the total amount or add all the total amounts together and then multiply that total by 0.2? Use your method to find the total tip amount.,

Both methods will result in the same answer $206.50

To estimate the width of a firework burst, use the formula $w = 5.7At$
A is the estimated viewing angle of the display, t is the time (in seconds) from the instant you see light until you hear the sound and w is the width (in metres). James is 2 km from a fireworks display and counts 5 seconds between seeing the light and hearing the explosion sound of the fireworks. He estimate the viewing angle at 5°. Use this information to estimate the width of the firework's burst. $5.7 \times 5° \times 5 = 142.5 \text{ m}$

1 Put a +, -, ×, or ÷ sign into each ☐ to make the equations correct.

$5 \boxed{+} (3 + 5) = 13$

$9^2 - 9^2 = 8 \boxed{-} 8$

$\sqrt{100} \boxed{\div} (2 \times 5) = 1$

$2^4 + 2^3 = 8 \boxed{\times} 3$

$36 \boxed{+} (\sqrt{16} \times 3) = 48$

$5! \boxed{\div} 3 = 40$

$4! \boxed{\times} (12 - 3!) = 144$

$3 \times \sqrt{100} = 3! \boxed{\times} \sqrt{25}$

$\dfrac{200}{4 \boxed{\times} 2} = 25$

$\dfrac{27 \boxed{+} 33}{22 \boxed{-} 18} = 15$

2 Reynolds has 4 class tests in one week. He receives the following marks:
17 out of 25 for Science,
29 out of 40 for English,
10 out of 15 for History
and 25 out of 30 for Maths.

Write all these marks as percentages and rank her 4 marks from highest down to the lowest.

Rank		Percentage
3	Science	68%
2	English	72.5%
4	History	66.7%
1	Maths	83.3%

3 Evaluate the expressions when x = 20, y = 5

$0.5x + y =$ _____ 15 _____

$\dfrac{x + 5}{y} =$ _____ 5 _____

$0.3(x - y) =$ _____ 4.5 _____

$y^2 - 2x =$ _____ -15 _____

4
The ratio of the sides of the triangle are written beside each of the sides. If the perimeter of the triangle is 60 cm then give the length of each side.

7

3

5

3:5:7 = 15 parts. 60cm ÷ 15 = 4 cm

This means each part is 4cm,

3:5:7 = sides of 12, 20, 28 cm

5 A bakery sells 1 dozen croissants for $5.40, How much would it cost for 15 croissants?

$5.40 ÷ 12 = 0.45 per croissant

0.45 × 15 = $6.75 for 15 croissants

BLOODY MATHS!

1. The picture shows blood through a microscope.
   Red blood cells have a diameter of 7.0×10^{-6} m.
   White blood cells have a diameter of 1.2×10^{-5} m.
   How many times wider are white blood cells than red blood cells?

   $1.2 \times 10^{-5} \div 7.0 \times 10^{-6} = 1.71$ times wider
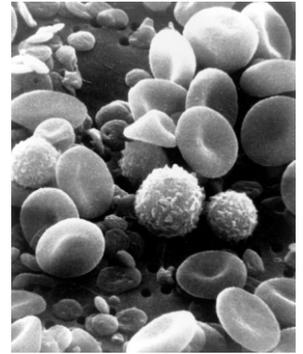
The diagram below shows the blood types of donors going to a blood bank.
45% of the people going to give blood are Type O.
35% of the people going to give blood are Type A.
15% of the people going to give blood are Type B
5% of the people going to give blood are Type AB
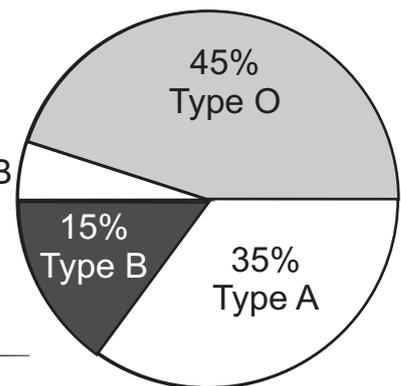
2. Patients with Type A blood can also
   accept Type O blood in a transfusion.
   Give the probability that a randomly selected
   donor will be either Type A or Type O.

   $35\% + 45\% = 80\%$

3. Patients with Type B blood can also accept Type O blood in a transfusion. Give
   the probability that a randomly selected donor will be either Type B or Type O.

   $15\% + 45\% = 60\%$

4. What is the probability that a randomly selected donor does not have Type O
   blood?

   $100\% - 45\% = 55\%$

5. What is the probability that a randomly selected donor does not have Type A or
   Type B blood.

   (Type O) $45\% +$ (Type AB) $5\% = 50\%$

6. Two donors come into the centre at the same time. What is the probability that
   both are blood Type O?

   $45\% \times 45\% = 20.25\%$

7. There are 400 donors who go to the blood bank in a week. Each donate 470 ml
   of blood. How much of the blood type AB would you expect in 1 week.

   $400 \times 5\% = 20$ people

   20×470 ml $= 9400$ ml

   $= 9.4$ litres

TRIANGLES FULL OF ODD NUMBERS
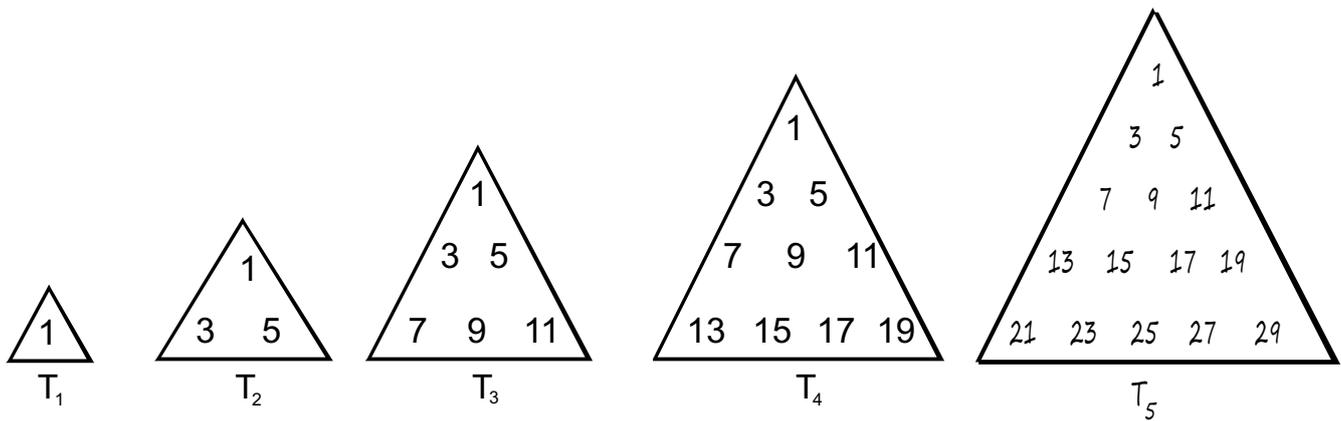
In the diagram above, the set of odd numbers, starting at 1, are placed into triangles of different sizes. The first 4 triangles are shown.

1. In the unmarked triangle above, draw T5, the 5th triangle of odd numbers.

2. The table below gives the sum of the numbers in the bottom row of each triangle. The variable n gives the triangle number. Complete the table

Triangle	T1	T2	T3	T4	T5	Tn
Sum of bottom row	1	8	27	64	125	n^3

3. What would the sum of the numbers in the bottom row of the 100th triangle equal?

$$100^3 = 1\ 000\ 000$$

4. The sum of the numbers in the bottom row of the nth triangle of odd numbers is 1 728. Find the value of n.

$$Tn = 1728, \therefore n = \sqrt[3]{1728}, \quad n = 12$$

5. Calculate the triangle number whose bottom numbers sum close to 3300.

The cube root of the sum must equal a whole number, $\sqrt[3]{3300} = 14.89$

The nearest triangle number is 15, $\therefore 15^3 = 3375$

6. Claudia has come up with an expression to calculate the sum of all the numbers in each of the triangles. Her expression is: sum $= \left(\dfrac{n}{2} + \dfrac{n^2}{2}\right)^2$

Use her rule to complete the table below. Does the rule seem correct?

Triangle	T1	T2	T3	T4	T5
Sum of bottom row	1	9	36	100	225

The rule seems to be correct (it is correct for the first 5 triangles)

BLACK Worksheet 17

A ROYAL DILEMMA

The Queen wants to divide 11 horses in her stable amongst her 3 children.

The oldest child is to receive $\frac{1}{2}$ of the horses, the second child is to receive $\frac{1}{4}$ of the horses and the youngest is to receive $\frac{1}{6}$ of the horses.

The lawyers are at the stables but cannot think of a way to do this. The Queen then rides up on her horse and says "I have arrived and I have a solution to the problem." What could his solution possibly be?

The Queen can include her 12th horse in the calculations.

½ of 12 is 6 horses

¼ of 12 is 3 horses

⅙ of 12 is 2 horses

There are now a total of 11 horses allocated and the Queen

can still ride off on her 12th horse.

FACTORISING TRINOMIALS

Solve each equation by factorising

1. $x^2 + 12x + 35 = 0$

   $(x + 7)(x + 5) = 0$

   $x = -7, x = -3$

2. $x^2 - x - 20 = 0$

   $(x - 5)(x + 4) = 0$

   $x = 5, x = -4$

3. $x^2 + 9x - 22 = 0$

   $(x + 11)(x - 2) = 0$

   $x = -11, x = 2$

4. $x^2 - 18x + 80 = 0$

   $(x - 10)(x - 8) = 0$

   $x = 10, x = 8$

5. $x^2 + 15x = -50$ $x^2 + 15x + 50 = 0$

   $(x + 10)(x + 5) = 0$

   $x = -10, x = -5$

6. $x^2 - 5x = 24$ $x^2 - 5x - 24 = 0$

   $(x - 8)(x + 3) = 0$

   $x = 8, x = -3$

A GROWING TREND

Flora is a botanist and she has been studying the rare Carpen Tree. She found her first Carpen Tree in 2002. The graph below shows the height of this tree, measured annually since 2002. A growth trend has been added to the graph.
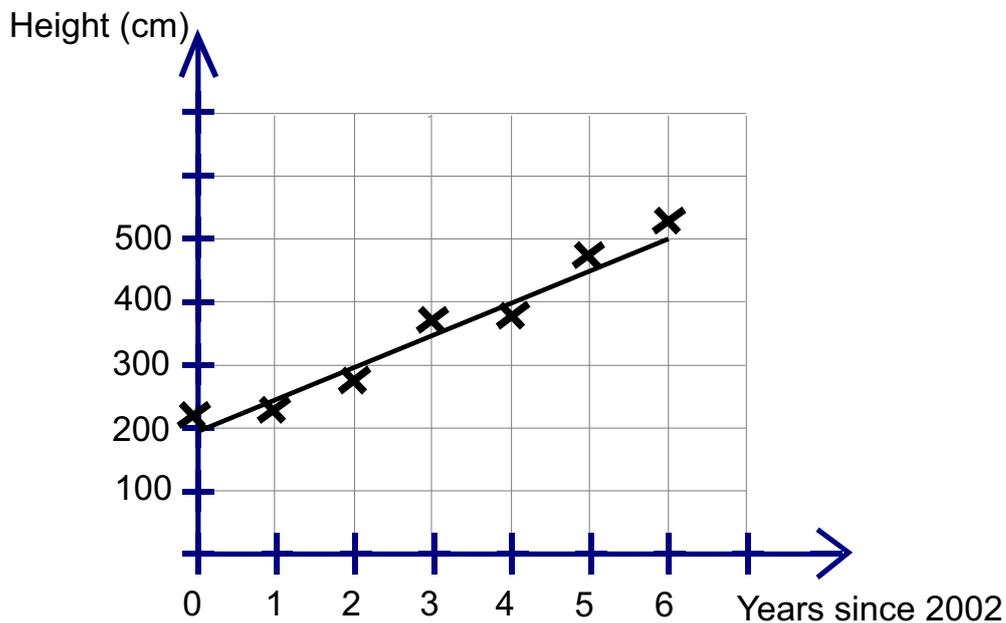
1. Find the equation of the growth trend where: y = the height of the tree, in cm and x = the number of years since 2002.

   gradient (slope) = 50, y intercept = 200

   ∴ using y = mx + c, equation is y = 50x + 200

2. What does the gradient of the trend line show about how this tree is growing?

   The gradient (rate of growth) shows that during each year, the tree is growing (approx) 50 cm

3. Flora estimates that the tree was about 4 years old when she found it. Why is this a reasonable estimate?

   Using a gradient (growth rate) of 50 cm means that it would have taken 4 years to reach 200 cm

4. In what year do you estimate that the tree will first reach 10 metres tall?

   Use the equation y = 50x + 200 and 1000 cm = 10 metres

   1000 = 50x + 200

   800 = 50x

   x = 16 years from 2002 (2018)

EXPANDING
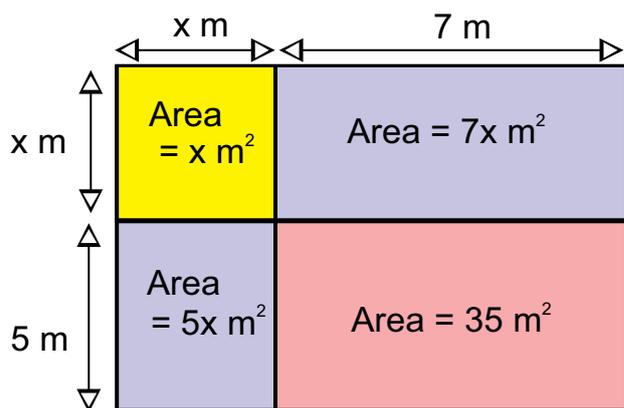
multiplying $(x + 7)(x + 5)$

$= x^2 + 5x + 7x + 35$

$= x^2 + 12x + 35$

Expand and simplify.

$(x + 3)(x + 4)$

$= x^2 + 4x + 3x + 12$

$= x^2 + 7x + 12$

$(x + 5)(x + 3)$

$= x^2 + 3x + 5x + 15$

$= x^2 + 8x + 15$

$(x + 7)(x - 3)$

$= x^2 - 3x + 7x - 21$

$= x^2 + 4x - 21$

$(x - 6)(x - 4)$

$= x^2 - 4x - 6x + 24$

$= x^2 - 10x + 24$

$(x - 7)(x + 3)$

$= x^2 + 3x - 7x - 21$

$= x^2 - 4x - 21$

$(x + 4)(x + 2)$

$= x^2 + 2x + 4x + 8$

$= x^2 + 6x + 8$

$(x + 6)(x - 2)$

$= x^2 - 2x + 6x - 12$

$= x^2 + 4x - 12$

$(x + 4)(x - 5)$

$= x^2 - 5x + 4x - 20$

$= x^2 - x - 20$

$(x - 9)(x - 2)$

$= x^2 - 2x - 9x + 18$

$= x^2 - 11x + 18$

$(x - 4)(x + 1)$

$= x^2 + x - 4x - 4$

$= x^2 - 3x - 4$

WET MATHEMATICS

The graph below represents a swimming race between Edith and John.
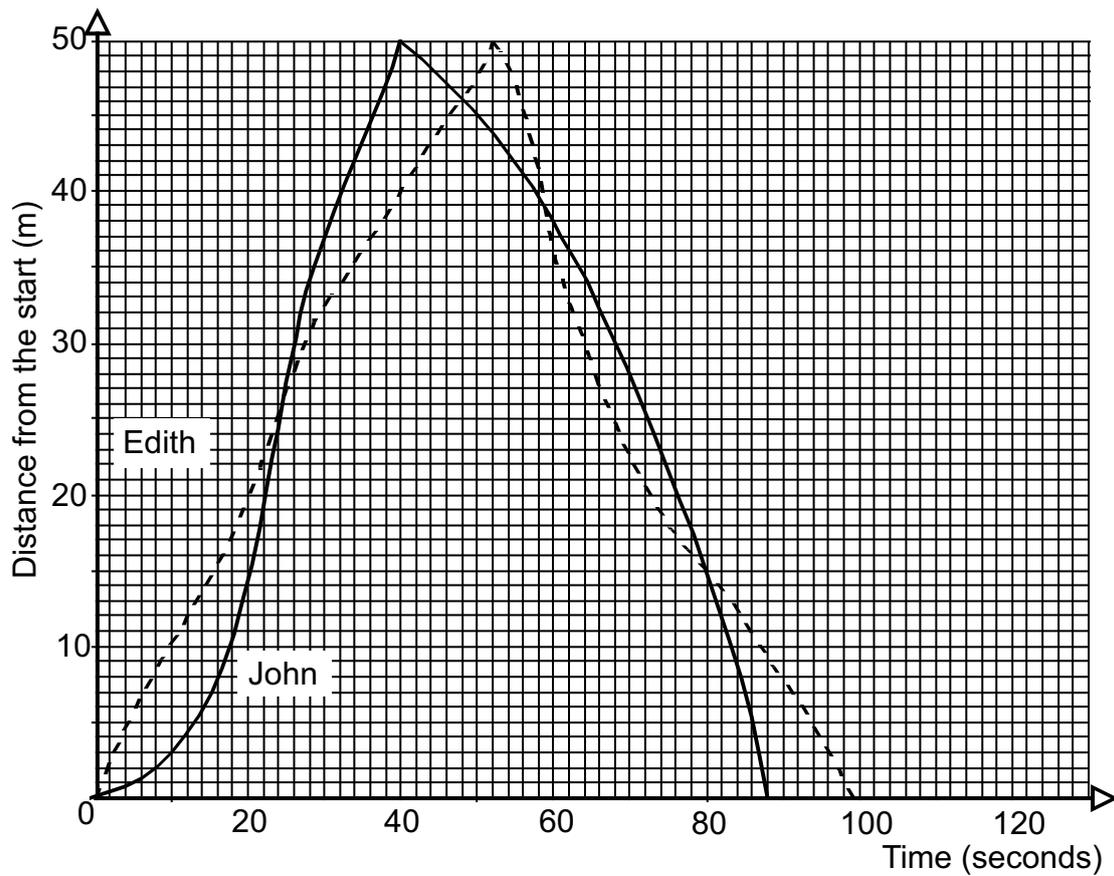
1. At what time did John overtake Edith for the second time?

   80 seconds

2. Give the maximum distance between the swimmers during the race

   10 metres (at 40 seconds)

3. Who was swimming faster at 56 seconds? How can you tell?

   Edith – a steeper slope on her graph

THE UPS AND DOWNS OF MATH

1. An amusement park roller coaster ride includes a free fall drop of 90 metres. Engineers use the equation $d = 5t^2$ to determine the time (t seconds) that it takes the roller coaster car to travel a distance of d metres. How long will it take to travel the 90 metres?

   $d = 5t^2$, therefore $90 = 5t^2$

   $18 = t^2$

   $t = \pm 4.2$ seconds

   $= 4.2$ seconds (only the positive solution makes sense in this situation).

BLACK Worksheet 18

PERCENTAGE APPLICATIONS

The Prime Minister has just called you on the phone. Apparently GDP (Gross Domestic Product) rose 5% last month but fell 30% this month. He wants to know what the overall percentage change is over the last two months. Calculate the answer for him.

A rise of 5% is 1.05 × the total. A fall of 30% is 0.7 × the total

1.05 × 0.7 = 0.735 and 1 – 0.735 = 0.265

Therefore tell the Prime Minister that GDP fell by 26.5% over 2 months.

NUMBER PRACTICE

1. $2(6^2 - 9) =$ _____ 54 _____

2. $\frac{1}{3}(7^2 - 4) =$ _____ 15 _____

3. $[38 - (8 + 3)] \div 3 =$ _____ 9 _____

4. $\frac{1}{2}[28 + 5(-2)] =$ _____ 9 _____

5. $2 + 8(5) \div 2 + 3 =$ _____ 25 _____

Let $w = 6$, $x = 0.4$, $y = \frac{1}{2}$ and $z = -2$

Evaluate each expression

6. $w \times x \times z$ _____ –4.8 _____

7. $w + 20 \div y$ _____ 46 _____

8. $y^2 + w$ _____ 6.25 _____

9. $\frac{-2z - 15x}{3y}$ _____ –1.33 _____

10. $(x - y)^2$ _____ 0.01 _____

REAL LIFE SITUATIONS

Many athletes suffer concussion as a result of a sports collision or injury. The graph below shows the relationship between blood flow to the brain and the number of days after the concussion.

Blood Flow After Concussion

(graph: Percent of blood flow (y-axis: 0, 25%, 50%, 75%, 100%) vs Days (x-axis: 1–8), curve rising from 50% to 100%)
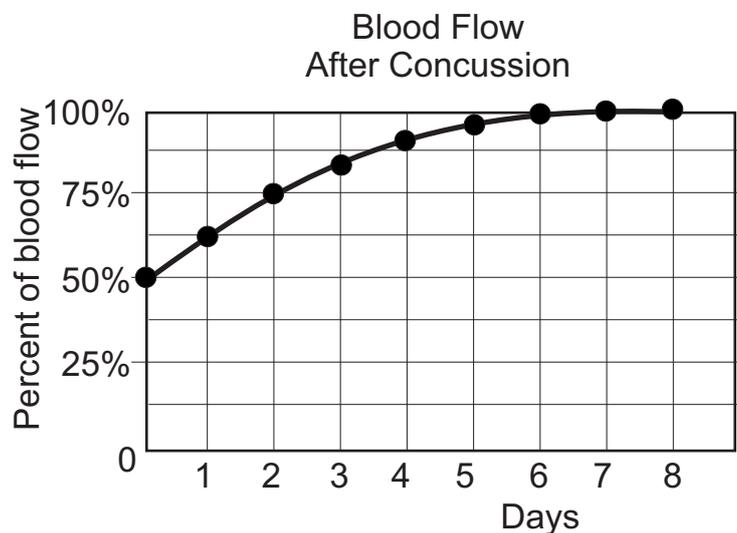
Describe the graph and comment on the statement "Athletes should have two weeks rest after a concussion."

After a concussion the blood flow returns to normal after 6–7 days. Because of individual differences it would be best to wait another 7 days before any sport.

BUILDING WITH MATHEMATICS

This diagram is a plan view of a garage and workshop.
All dimensions are in metres.

1. Calculate the missing dimensions.

AB = ___AB = 4.5m___ m

BC = ___BC = 4m___ m
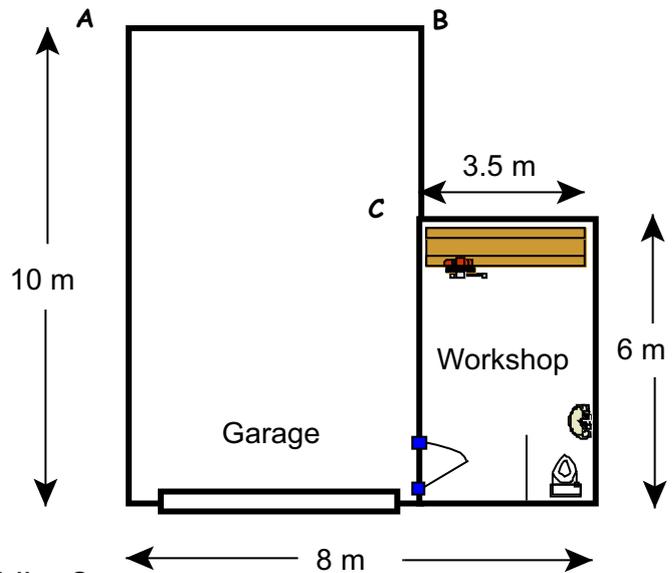
2. What is the total perimeter of the building?

___Perimeter = 36m___

___(10 m + 10 m + 8m + 8m)___

3. What is the total area of the building?

___Total Area = 66m²___

___(10 × 4.5 + 6 × 3.5)___

The diagram below shows the front view of the garage and the workshop.
The roof forms an isosceles triangle with the rest of the building.

4. What is the area of the isosceles triangle that
is formed (DEF)?

___Area △DEF = 3.4m²___

___(Area ½ × 8 × 0.85)___

Front view of garage

5. What is the length of the roof marked DE?

___DE = 4.09m___

___Use Pythagorus DE² = (0.85)² + (4)²___
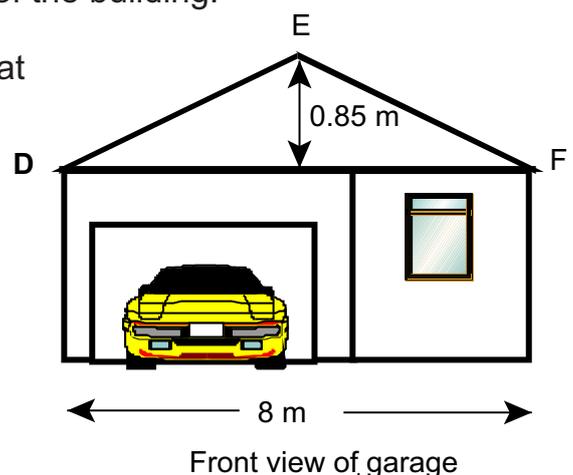
6. What is the angle of slant on the roof (EDF and EFD)?

___Angle = 12°___

___Use Tan⁻¹ D = $\frac{0.85}{4}$___

PLOTTING PARABOLAS

1. Factorise x^2 - 5x - 24 $\underline{(x - 8)(x + 3)}$

2. Solve the equation x^2 - 5x - 24 = 0 $\underline{x = 8 \ or \ x = -3}$

3. The sketch below shows the graph of y = x^2 - 5x - 24. The curve forms a parabola and cuts the x axis at the points A and B. The curve also cuts the y axis at point C. Write down the co-ordinates of A, B and C.

    <u>Using 1. and 2 points A and B are</u>

    <u>(-3, 0) and (8, 0)</u>

    <u>C cuts the axis where x = 0</u>

    <u>Therefore putting x = 0 into the</u>

    <u>equation then C = (0, -24)</u>
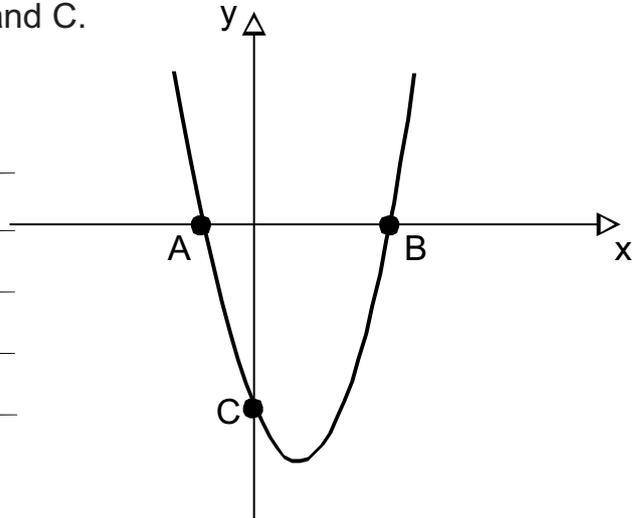
4. The graph of the equation y = x^2 - 4x - 1 forms a curve called a parabola.
   Complete the table of values below.
   Use these values to sketch the graph of y = x^2 - 4x - 1 on the grid below.

x	-1	0	1	2	3	4	5
x^2	1	0	1	4	9	16	25
- 4x	+ 4	0	- 4	- 8	-12	-16	- 20
- 1	-1	-1	-1	-1	-1	-1	-1
y	4	-1	-4	-5	-4	-1	4

5. Draw the straight line represented by y = 2x - 6. Write the intersection points of the parabola with this equation

    $y = x^2 - 4x - 1$ $y = 2x - 6$

    <u>Intersection points (1, -4) and (5, 4)</u>

6. The two graphs intersect where x^2 - 4x - 1 = 2x - 6.
   Solve the equation to see if your answer to 5. (above) is correct.

    $\underline{x^2 - 6x + 5 = 0}$

    $\underline{(x - 5)(x - 1) = 0}$

    $\underline{x = 5, x = 1}$

    <u>These are the same x points as 5.</u>
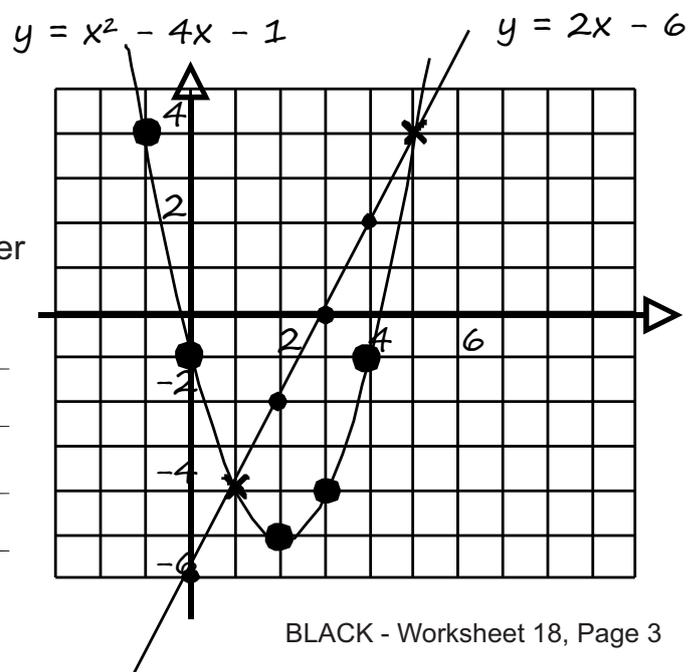
EXPANDING

Expand and simplify the following

$(x + 7)(x + 3)$

$x^2 + 3x + 7x + 21$

$x^2 + 10x + 21$

$(x + 6)(x + 4)$

$x^2 + 4x + 6x + 24$

$x^2 + 10x + 24$

$(x + 6)(x + 2)$

$x^2 + 2x + 6x + 12$

$x^2 + 8x + 12$

$(x + 5)(x + 3)$

$x^2 + 3x + 5x + 15$

$x^2 + 8x + 15$

$(x + 3)(x - 2)$

$x^2 - 2x + 3x - 6$

$x^2 + x + - 6$

$(x + 4)(x - 1)$

$x^2 - x + 4x - 4$

$x^2 + 3x - 4$

$(x + 9)(x - 3)$

$x^2 - 3x + 9x - 27$

$x^2 + 6x - 27$

$(x + 7)(x - 6)$

$x^2 - 6x + 7x - 42$

$x^2 + x - 42$

$(x - 5)(x + 3)$

$x^2 + 3x - 5x - 15$

$x^2 - 2x - 15$

$(x - 8)(x + 5)$

$x^2 + 5x - 8x - 40$

$x^2 - 3x - 40$

$(x - 10)(x + 6)$

$x^2 + 6x - 10x - 60$

$x^2 - 4x - 60$

$(x - 9)(x + 2)$

$x^2 + 2x - 9x - 18$

$x^2 - 7x - 18$

$(x - 4)(x - 3)$

$x^2 + 3x - 4x + 12$

$x^2 - x + 12$

$(x - 5)(x - 2)$

$x^2 - 2x - 5x + 10$

$x^2 - 7x + 10$

$(x - 8)(x - 4)$

$x^2 - 4x - 8x + 32$

$x^2 - 12x + 32$

$(x - 4)(x - 2)$

$x^2 - 2x - 4x + 8$

$x^2 - 6x + 8$

$(x + 4)(x - 4)$

$x^2 - 4x + 4x - 16$

$x^2 - 16$

$(x + 10)(x - 10)$

$x^2 - 10x + 10x - 100$

$x^2 - 100$

$(x + 5)^2$

$x^2 + 5x + 5x + 25$

$x^2 + 10x + 25$

$(x + 8)^2$

$x^2 + 8x + 8x + 64$

$x^2 + 16x + 64$

$(x - 7)^2$

$x^2 - 7x - 7x + 49$

$x^2 - 14x + 49$

BLACK Worksheet 19

NUMBER CALCULATIONS

1. John invests $610 into a bank account paying compound interest at a rate of 6.5% per annum. By completing the spaces below, calculate the total amount in John's account after 2 years. Round your final answer to 2 decimal places.

   Starting Amount: $ _$610_ × _1.065_ = Year 1 Total Amount: $ _649.65_

   Year 1 Amount: $ _$649.65_ × _1.065_ = Year 2 Total Amount: $ _691.88_

The heat setting of a gas oven is called its Gas Mark. A Gas Mark, G, can be converted to a temperature, C° by using the formula: C = 15G + 120

2. Factorise the expression 15G + 120.

$$15G + 120 = 15(G + 8)$$

3. Make G the subject of the formula C = 15G + 120.

$$C - 120 = 15G$$
$$C = \frac{C - 120}{15}$$

The distance from the Earth to the Moon is 400 000 kilometres.

4. Write the number 400 000 in standard form.

$$400\ 000 = 4.0 \times 10^{5}$$

5. The distance from the Earth to the Sun is 1.5×10^{8} kilometres.

   Calculate the value of the expression: $\dfrac{\text{distance from the Earth to the Moon}}{\text{distance from the Earth to the Sun}}$

   Give your answer in standard form.

$$\frac{4.0 \times 10^{5}}{1.5 \times 10^{8}}$$

$$= 2.7 \times 10^{-3} \text{ (1 DP)}$$

HOSPITAL MATH

Chemists studying the use of aspirin have found that the amount of aspirin (in milligrams) in your bloodstream after t hours is given by the function $y = A(0.7)^t$ where A is the initial amount of aspirin in a given dose.

1.   A doctor prescribes 2 aspirin tablets every 4 hours. Each tablet has 200 mg of aspirin. How much aspirin will still be in the bloodstream after 8 hours?

   Hour 0 (initial dosage) = 400 mg

   Hour 1: $400(0.7)^1$ = 280 mg

   Hour 2: $400(0.7)^2$ = 196 mg

   Hour 3: $400(0.7)^3$ = 137.2 mg

   Hour 4: $400(0.7)^4$ = 96.04 mg

   At this point the patient takes another 200 mg

   Hour 5: $400(0.7)^1 + 400(0.7)^5$ = 280 mg + 67.2 mg = 347.2 mg

   Hour 6: $400(0.7)^2 + 400(0.7)^6$ = 196 mg + 47.0 mg = 243 mg

   Hour 7: $400(0.7)^3 + 400(0.7)^7$ = 137.2 mg + 32.9 mg = 170.1 mg

   Hour 8: $400(0.7)^4 + 400(0.7)^8$ = 96.0 mg + 23.0 mg = 119

A car and an ambulance are traveling towards each other. They are 1 km (1000 metres) apart. The car is traveling at a speed of 50 km / hr (13.9 metres / second). The ambulance is traveling faster at 80 km / hr (22.2 metres / second). The ambulance is racing to an accident and has its siren on. In ideal conditions, the siren can be heard from 400 m.

2.   If the conditions are ideal, in how many seconds will the driver of the car first hear the siren?

| Ambulance 22 m/s | → | siren (400 m) | ← | Car 13.9 m/s |

1000 metres

   1000 – 400 = 600 (this is the total distance that needs to be

   traveled by both cars before the siren is heard.

   Distance traveled by car = 13.9t

   Distance traveled by ambulance = 22.2t

   Therefore 13.9t + 22.2t = 600

   36.1t = 600

   t = 16.6 seconds

MATHS IN SPACE

The book "Rocket Boys" was made into a movie called "October Sky". It is based on the true story of a coal miner's son who takes up rocketry and eventually becomes a NASA scientist. In the story he experiments with different rocket designs.

He uses a formula $R = \dfrac{S + F + P}{S + P}$

R is the mass ratio of a rocket.
S is the mass of the structure.
F is the mass of the fuel.
P is the mass of the payload (that part of the rocket such as satellites etc that are being carried for a fee into space).

A more efficient rocket design requires less propellant to achieve a given goal, and would therefore have a lower mass ratio. A higher mass ratio permits the rocket to achieve a higher velocity. Typical rockets have mass ratios in the range from 8 to 20. The Space Shuttle, for example, has a mass ratio of 16.

A small rocket is designed with a mass structure of 900 grams and a payload of 1500 grams. How much fuel (in kilograms) should be loaded into the rocket if it is to have a mass ratio of 8?

$$R = \frac{S + F + P}{S + P}$$

$$8 = \frac{900 + F + 1500}{900 + 1500}$$

$$8 = \frac{2400 + F}{2400}$$

$$8 \times 2400 = 2400 + F$$

$$19\,200 = 2400 + F$$

$$F = 16\,800 \text{ grams,}$$

i.e. 16.8 kg of fuel should be loaded.

SICK MATHEMATICS

A nurse dispensing medication has to take into account the weight of the patient. In one particular instant, a patient is taking 150 milligram blood pressure tablets. The dosage is 5 milligrams per kilogram of body weight and the tablets should be taken 3 times a day (every 8 hours). If the patient weighs 120 kg, how many tablets would be needed for a 30 day supply?

120 kg × 5 milligrams per kg = 600 milligrams

This means 600 milligrams ÷ 150 milligrams = 4 tablets

4 tablets, 3 times a day for 30 days is 4 × 3 × 30

= 360 tablets

MATHEMATICS IN DESIGN

The diagram represents the plan view of the corner of a display cabinet. The corner is fitted with a concave piece of glass (represented by the arc AB).

The arc AB is a quarter of a circle with the centre of the circle at C. The circle radius is 46cm.
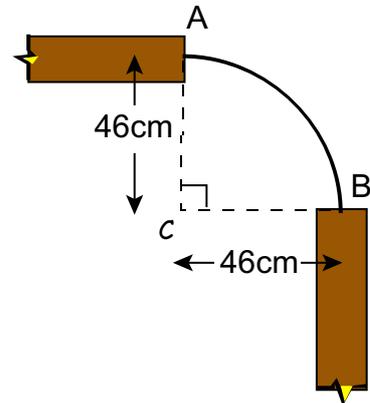
Note: diagram is not drawn to scale

1.   Calculate the length of the arc AB.
      Give your answer correct to 3 significant figures.

Circumference = πD

For this example the diameter = 92cm and you must × by 1/4

AB = 72.3 cm (3 sf)

In another design, the concave window in part 1. is replaced by 3 identical glass panels. These panels are all straight and are represented in the diagram below by the lines AP, PQ and QB.

2.   Calculate the length of each panel
      (i.e. calculate the lengths of one of AP, PQ or QB).
      Give your answer correct to 3 significant figures.

Note: diagram is not drawn to scale
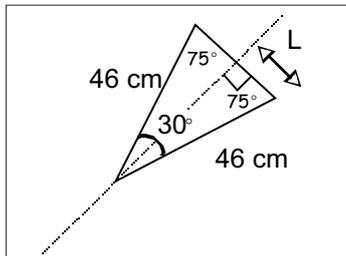
$Cos\ 75 = \dfrac{L}{46}$ L = 11.91 cm

∴ the window panel (2 × 11.91)

= 23.8 cm (3 sf)
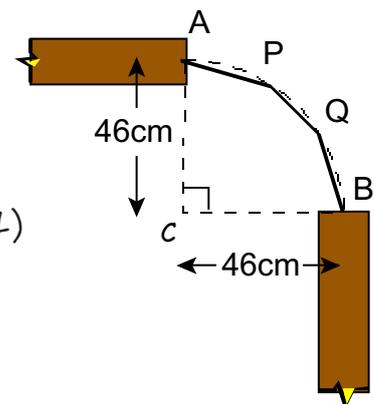
6.   The Pyramid Arena is a sports stadium in Memphis, USA. It is the 6th largest pyramid in the world.

It has a 183 metre square base and is 98 metres tall.

What is the volume of this pyramid?

$V = \dfrac{1}{3}Ah$

$= \dfrac{1}{3} \times 183 \times 183 \times 98$

$= \dfrac{1}{3} \times 183 \times 183 \times 98$

= 1 093 974 m²

BLACK Worksheet 20

MATHEMATICAL APPLICATIONS

1. On July 20, 1969, Neil Armstrong stepped onto the surface of the moon. On the moon his space suit and life support packs weighed only 15 kg. However the weight of anything on the moon is only one sixth the weight on earth. What would be the weight of his space suit and life support packs on earth?

   one sixth on the moon means 6 times that mass on earth

   6 × 15kg = 90 kg

2. A flock of snow geese was tracked on one particular migration flight. The geese managed to fly 600 km in 7.5 hours. Write an equation so that the distance flown can be worked out for any given time.

   Distance travelled = speed × time

   Using the data above

   600 km = s × 7.5 hours

   therefore s = 80 km / hour

   This means the equation is D = 80t where D = distance travelled

   and t = time in hours.

3. The intensity of sound can be measured in watts per square metre. The table below gives watts per square metre for some common sounds.

10^2	jet plane engine(30 metres away)
10^1	pain level
10^0	amplified rock music (2 metres away)
10^{-2}	a noisy kitchen / classroom
10^{-3}	heavy traffic
10^{-6}	normal conversation
10^{-7}	average home
10^{-9}	whisper
10^{-12}	barely audible sound

   How does the intensity of a whisper compare to that of a normal conversation?

   Whisper = 10^{-9}, normal conversation = 10^{-6}

   A whisper is 1000 (10^3) times quieter

THE PERCENTAGE GAME

1 A concert ticket costs $154. GST of 15% is added to this. What is the total price for 4 tickets?

$154 × 1.15 = $177.10

$177.10 × 4 = $708.40

2 A shirt is on sale for 35% off the original price. If the original price was $79 what is the discounted price?

$79 × 0.65 = $51.35

3 According to the figures released by a job website, the average starting income of a university graduate is $49 478. The average starting income of a person with just a NCEA qualification is $24 395. Calculate the percentage increase in the starting income for a person who leaves school with NCEA and another who goes and earns a University degree.

Difference in starting incomes is: $49 478 – $24 395 = $25 083

Percentage difference is: $25 083 ÷ 24 395 = 1.028

Therefore percentage difference is 103%

4 In Chemistry class, Rebecca boiled 120 millilitres of solution for 10 minutes. Afterwards, only 111 millilitres remained due to evaporation. What was the percentage of decrease in the amount of liquid?

Difference in volume is: 120 – 111 = 9 millilitres

Percentage difference is: 9 ÷ 120 = 0.075

Therefore percentage decrease is 7.5%

5 Fuji wants to gain weight for wrestling. At present he is 90 kg but to wrestle in the heavy-weight class he wants to be 105 kg. What is the percentage of weight that Fuji needs to gain?

Difference in weight is: 105 kg – 90 kg = 15 kg

Percentage difference is: 15 ÷ 90 = 0.1666

Therefore percentage increase is 16.7%

6 Last year the Mahobe share price rose 32% in the first half of the year and then it rose a further 10% in the second half of the year. What was the overall percentage change?

Increase of 32% = 1.32 × total, increase of 10% = 1.1 × total

Therefore 1.32 × 1.1 = 1.452

= increase of 45.2%

APPLYING MATHEMAICS

Pressure is commonly measured in "pounds per square inch" (PSI).

The formula $P = \dfrac{1.2W}{H^2}$ represents the amount of pressure exerted on the floor by the heel of a shoe.

> P represents the pressure in pounds per square inch (PSI)
> W represents the weight of a person wearing the shoe (in pounds)
> H represents the width of the heel of the shoe (in inches)

1. Find the weight of the person (in kg) if the heel is 75 mm wide and the pressure exerted is 25 pounds per square inch. Note - you need to convert mm to inches (25 mm = 1 inch) and investigate how many pounds in a kilogram.

$$75 \text{ mm} = 3 \text{ inches}, \quad 25 = \frac{1.2 \times W}{3^2}$$

$$25 \times (3^2) = 1.2W$$

$$\frac{225}{1.2} = W$$

$$W = 187.5 \text{ pounds}$$

1 pound = 0.454 kilograms, therefore W = 85 kg

2. When exercising, a person's pulse should not exceed a certain limit depending on that person's age. The maximum rate is represented by the expression: P = 0.8(220 - a) where a is the age in years. Tim is advised by his trainer that his maximum pulse rate should not exceed 152. What is Tim's age?

$$152 = 0.8(220 - a)$$

$$152 = 176 - 0.8a$$

$$-24 = -0.8a$$

$$a = 30 \text{ years old}$$

3. Exercise physiologists have found that the formula $L = 0.0625s^2$ can approximately determine the maximum running speeds (in metres per second) that a person with leg length L (in metres) can sustain. Find the maximum running speed of a person with leg length 0.68 metres.

$$0.68 = 0.0625 \times s^2$$

$$0.68 \div 0.0625 = s^2$$

$$s^2 = 10.88, \; s = 3.3 \text{ m/sec}$$

USING ALGEBRA AND MEASUREMENTS

The diagram shows a wire box frame with square ends.
The frame is made using 100 cm of wire.

1. Form an equation, then find the value of x.

Equation is 4(x + 10) + 8X = 100

or 12x + 40 = 100

x = 5cm

x cm

(x + 10) cm

x cm
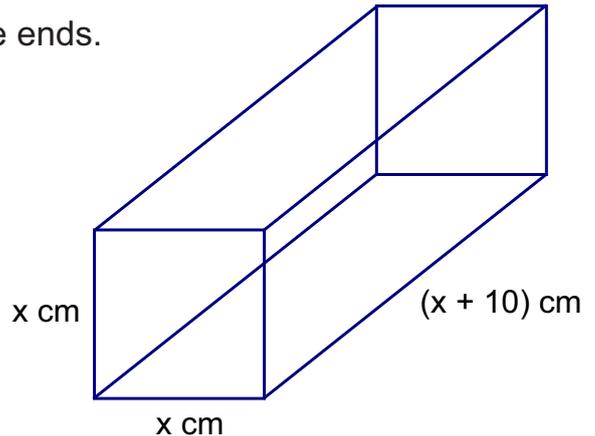
2. Calculate the volume of the box.

If x = 5 cm, Volume = 5 cm × 5cm × 15 cm

= 375cm^3

3. A 900 gram block of Edam cheese measures 18 cm × 9 cm × 5 cm.
Calculate the volume of the block of cheese.

18 × 9 × 5 = 810 cm^3

4. How many 900 gram blocks of cheese will fit in the carton shown? Most freight companies require the mass of a full carton to be under 25 kg. This make it easier to handle. Could the full carton of cheese meet this requirement?

Most packaging boxes allow for space.

The carton could have 2 × 4 blocks

of cheese on the base and be stacked

3 high

Therefore it could fit 24 blocks

EDAM CHEESE   HANDLE WITH CARE   16 cm

37 cm   37 cm

Weight of the cheese in the carton would be 24 × 0.9 kg = 21.6 kg

Given that a carton only weighs less than 0.5 kg it would easily

meet the mass requirement.

5. Edam cheese has 20% fat and Cheddar cheese has 35% fat. If 900 grams of Edam and 500 grams of Cheddar are blended together, what is the final percentage of fat in the final blended cheese?

Edam: 900 g × 0.2 = 180 g; Cheddar 500 g × 0.35 = 175 g

Therefore total fat is 180 + 175 = 355 grams (in 1400 grams total)

355 ÷ 1400 g = 25.4% fat in the final blended version.

BLACK Worksheet 21

If a = 48, b = 12, c = 7 and d = 4, calculate:

1. $\sqrt{a - b}$

$\underline{\quad 48 - 12 = 36 \quad}$

$\underline{\sqrt{36} = 6 \quad}$

2. $\sqrt{a + b + d}$

$\underline{\quad 48 + 12 + 4 = 64 \quad}$

$\underline{\sqrt{64} = 8 \quad}$

3. $-(c + d) + \sqrt{ab}$

$\underline{-(7+4) = -11, \ 48 \times 12 = 576}$

$\underline{-11 + \sqrt{576} = 13 \quad}$

4. $\sqrt{b^2 - (a + c + 2d)}$

$\underline{12^2 - (48 + 7 + 8) = 81}$

$\underline{\sqrt{81} = 9 \quad}$

5. $\sqrt{(a^2 + b^2 + c^2 + 3)}$

$\underline{\quad 2304 + 144 + 49 + 3 \quad}$

$\underline{= 2500 \quad}$

$\underline{\sqrt{2500} = 50 \quad}$

CHALLENGES

1. What is $\sqrt{500}$ to the nearest integer?

$\underline{\quad 22 \ (as \ 22^2 = 484) \quad}$

2. Solve $(x - 2)^2 + 1 = 37$

$\underline{\quad (x - 2)^2 = 36 \quad}$

$\underline{\quad x^2 - 4x + 4 = 36 \quad}$

$\underline{\quad x^2 - 4x - 32 = 0 \quad}$

$\underline{\quad (x - 8)(x + 4) = 0 \quad}$

$\underline{\quad x = 8 \ or \ x = -4 \quad}$
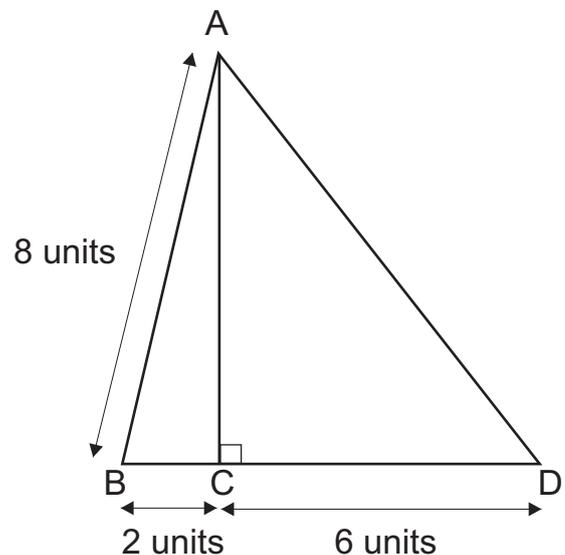
3. Find area of triangle ACD.

$\underline{\quad AC^2 = AB^2 - BC^2 \quad}$

$\underline{\quad AC^2 = 64 - 4 \quad}$

$\underline{\quad AC = 7.75 \quad}$

$\underline{\quad Area = \frac{1}{2} \times 6 \times 7.75 \quad}$

$\underline{\quad Area = 23.25 \ units^2 \quad}$

HANGING OUT

The table below gives the length of a spring when different masses are suspended from it.

Mass (grams), x	0	50	100	150	200
Length (mm), y	80	120	160	200	240

length (y)

mass (x)
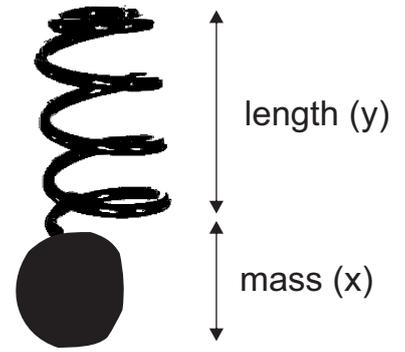
1.   Draw the graph of the table.

2.   Does the graph represent a linear function?

Yes. It is a straight line.

_____

3.   Write an equation for the line you have drawn.

$y = 0.8x + 80$

$y = $ *spring length*

$x = $ *mass*

Length of Spring
when different sized masses are attached

Length of Spring (mm)

200

100

0

50   100 150 200 250

Mass (grams)
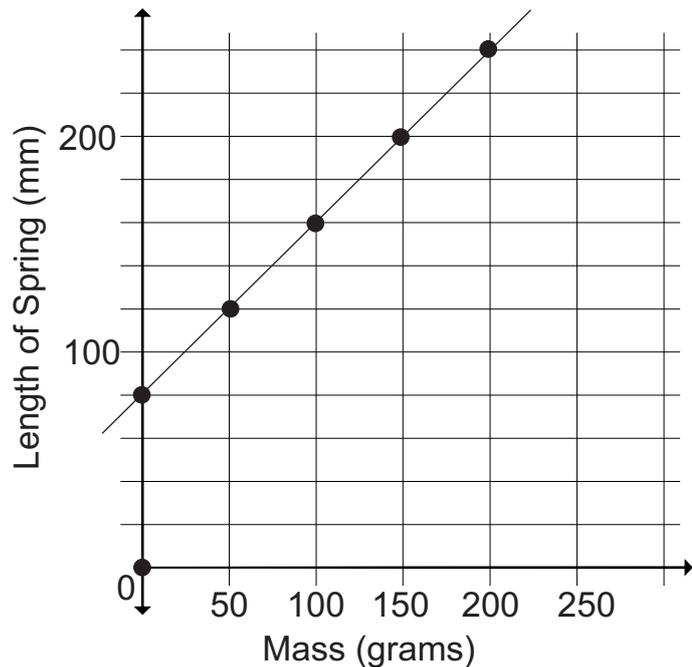
4.   If 500 grams was attached to the spring, what would be the length of the spring?

$y = 80 + 40x$, *therefore* $y = 80 + 0.8 \times 500$

spring length = 480 mm

WHAT A WASTE!

For Waikato counties with a population, p, the equation $W = 0.798p - 3785$ approximates the amount of solid waste, W, (in tonnes) that was disposed during 2010.

1.   The population of Hamilton is 140 700. To the nearest thousand tonnes, find how much solid waste was disposed of in Hamilton during 2010.

$W = 0.798 \times 140\ 700 - 3785$

$W = 108493.6$

$W = 108$ *(thousand tonnes)*

ROTATION, VECTORS and REFLECTION
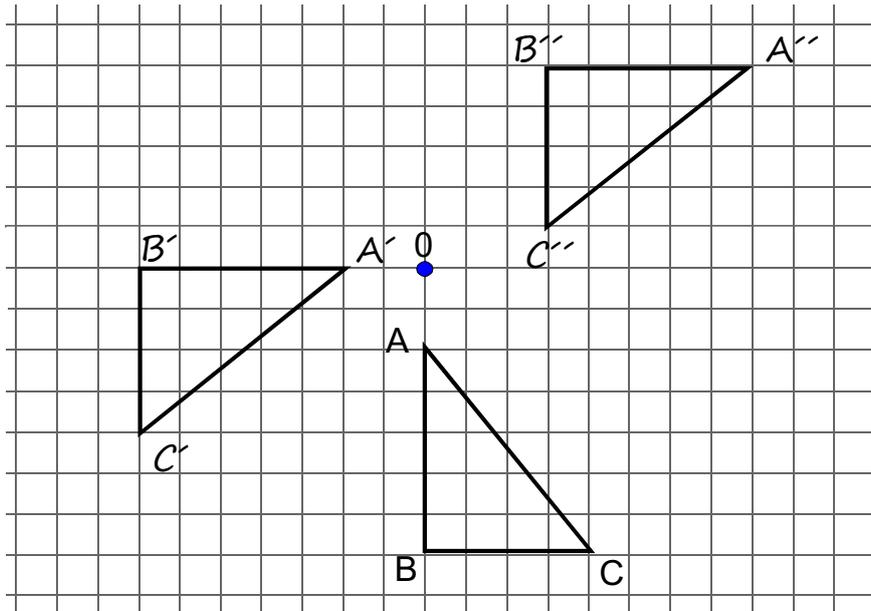
1. On the diagram above, draw the image of the triangle ABC after an anticlockwise rotation of 90° around point O.

2. Label your new image A'B'C'.

The image triangle A'B'C' is then translated by $\begin{pmatrix} 10 \\ 5 \end{pmatrix}$

3. Draw and label the resultant triangle A" B" C".
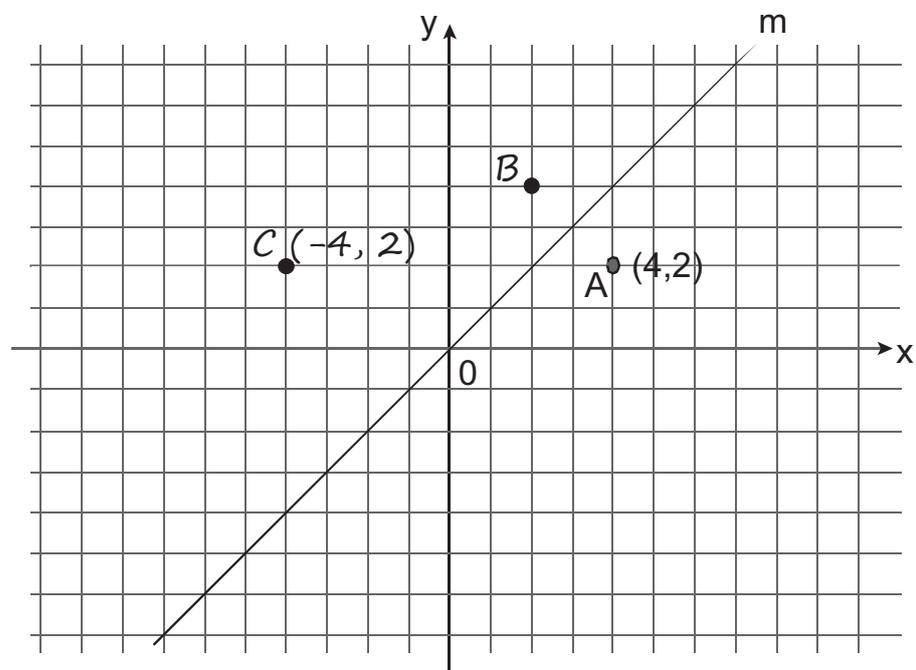
4. The graph shows the point A(4,2) and the line m (y=x). A → B under a reflection in the line m. Draw and label the point B on the graph.

5. B → C under a rotation of 90° anticlockwise, centre 0.
Write the co-ordinates of C beside your point.

6. Write the column vector for the translation B to C. $\begin{pmatrix} -6 \\ -2 \end{pmatrix}$

MONEY MATTERS

Edith is planning to go to university for 3 years. Each university semester is 20 weeks long and there are two semesters per year. To work out the costs over the 3 years she used a spreadsheet on her computer. She began to set up the table shown below.

	A	B	C	D
1	Study Costs Year 1	$4 500		3 year costs
2	Year 2	$5 000		
3	Year 3	$8 000	Total study costs	$17 500
4	Personal Costs (per year)	$6 000	Total personal costs	$18 000
5	Accommodation Costs (per week)	$ 190	Total accommodation costs	$22 800
6			Total cost over 3 years	

1. Write the formulas that Edith would use to calculate the values in Cells D3, D4 and D5 and D6.

   D3: =B1 + B2 + B3, or @sum(B1..B3)

   D4: =B4*3

   D5: =B5*120 (there are 40 weeks per year × 3 years study)

   D6: =D3 + D4 + D5, or @sum(D3..D5)

2. When Edith finishes her spreadsheet, what value should appear in Cell D6?

   $17 500 + $18 000 + $22 800 = $58 300

When Edith turned 10, her grandparents invested $15 000 in a bank savings account. After tax, the money has earned 4.5% compound interest per annum. Edith now uses a spreadsheet to calculate how much money is in the bank at the beginning of each year and the final amount that will be in the account when she turns 18.

	A	B	C
	Age (yrs)	Amount in the bank	Yearly Interest earned
1	10	$15 000.00	$675.00
2	11	$15 675.00	$705.38
3	12	$16 380.38	$737.12
4	13	$17 117.50	$770.29
5	14	$17 887.79	$804.95
6	15	$18 692.74	$841.17
7	16	$19 533.91	$879.03
8	17	$20 412.94	
9	18		

3. Calculate the values for Cells C8 and B9

   C9: $20 412.94 * 0.045 = $918.58

   C10: $20 412.94 + $918.58 = $21 331.52

BLACK Worksheet 22

FRACTIONS

1. What fraction of each shape is shaded?
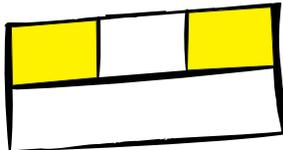
$\frac{2}{6} = \frac{1}{3}$
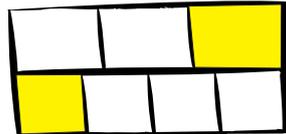
$\frac{1}{6} + \frac{1}{8} = \frac{7}{24}$
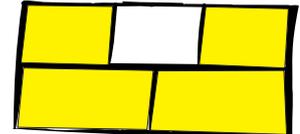
$\frac{2}{6} + \frac{2}{4} = \frac{5}{6}$
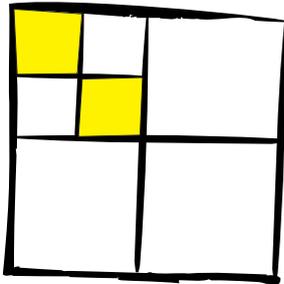
$\frac{2}{16} = \frac{1}{8}$
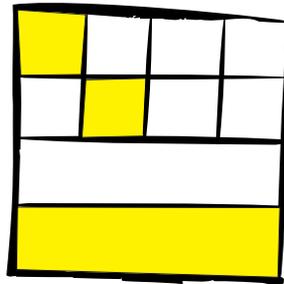
$\frac{2}{16} + \frac{1}{4} = \frac{3}{8}$
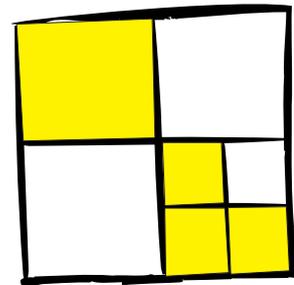
$\frac{1}{4} + \frac{3}{16} = \frac{7}{16}$

2. Look a the fraction division below and find the values for a and b.

$$\frac{19}{23} \div \frac{a}{b} = \frac{187\ 644}{28\ 382}$$

When dividing by a fraction you multiply by its reciprocal. $\frac{19}{23} \times \frac{b}{a}$

Therefore: $19b = 187644, b = 9876$

$23a = 28\ 382, a = 1234$

Fraction $\frac{a}{b} = \frac{1234}{9876}$

3. While using a calculator you press the add (+) button and add 36.5 instead of pressing the subtract (-) button and subtracting. Your incorrect answer is 118.25. What should the correct answer have been?

$a + 36.5 = 118.25, a = 81.75$

This is the figure before your mistake with the calculator keys.

Therefore $81.75 - 36.5 = 45.25$ (the answer you should have received)

TRANSFORMING

The diagram shows the vectors WX and YZ .

Centre of Rotation
(6, 7)
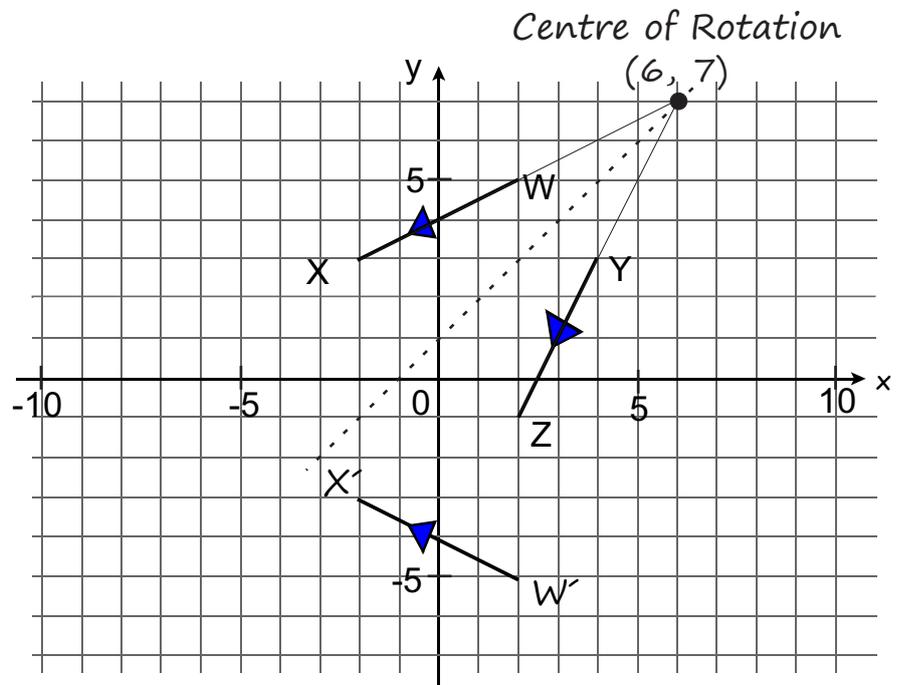
1. Write WX as a column vector. $\begin{pmatrix} -4 \\ -2 \end{pmatrix}$

2. WX is reflected in the y axis. Draw the resulting vector.

3. WX can be mapped on to YZ by a rotation.   *See answer on diagram*
Write the co-ordinates of the centre of rotation.

4. WX can also be mapped onto YZ by a reflection.
Draw a dotted line on the diagram to indicate the mirror line.

Triangle ABC (see below) has vertices at points (3,3), (5,3) and (7,5).

5. If the centre of enlargement is point (2,1) and the scale factor for enlargement is 3, sketch and label the new triangle A'B'C'.

6. Write the co-ordinates of the new triangle: *A′(5, 7), B′(11, 7), C′(17, 13)*

7. If the area of triangle ABC is 8 cm^2, what is the area of A'B'C'? *72 cm² (8× 3²)*

8. If Triangle ABC is enlarged with scale factor of -1, what is the area of the new triangle?

Area will still be 8 cm²
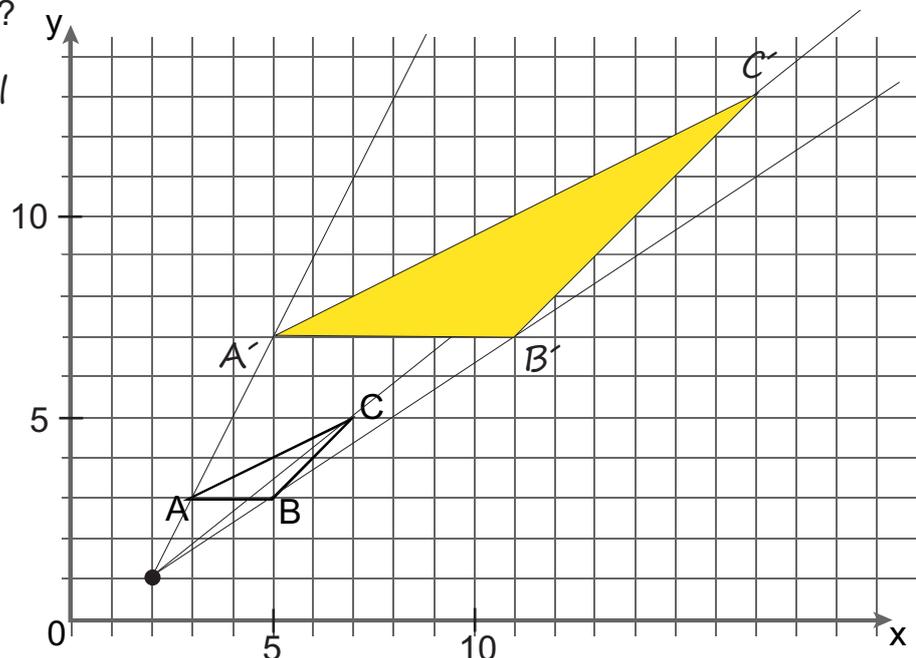
FACTORISING

Find the missing terms to complete each factorisation

$x^2 + 5x + 6$

$(x + 3)(x + 2)$

$x^2 + 7x + 12$

$(x + 4)(x + 3)$

$x^2 + 5x - 14$

$(x + 7)(x - 2)$

$x^2 + x - 20$

$(x + 5)(x - 4)$

$x^2 - 7x + 12$

$(x - 4)(x - 3)$

$x^2 - 10x + 24$

$(x - 6)(x - 4)$

$x^2 - x - 12$

$(x - 4)(x + 3)$

$x^2 - 4x - 5$

$(x - 5)(x + 1)$

$x^2 + 12x + 36$

$(x + 6)(x + 6)$

$x^2 + 4x + 3$

$(x + 3)(x + 1)$

$x^2 + 6x + 8$

$(x + 4)(x + 2)$

$x^2 + 3x - 28$

$(x + 7)(x - 4)$

$x^2 + 4x - 45$

$(x + 9)(x - 5)$

$x^2 - 8x + 16$

$(x - 4)(x - 4)$

$x^2 - 13x + 30$

$(x - 10)(x - 3)$

$x^2 - 4x - 21$

$(x - 7)(x + 3)$

$x^2 - 3x - 40$

$(x - 8)(x + 5)$

$x^2 - 25$

$(x - 5)(x + 5)$

Food (49%)

Paper (25%)

Plastic (9%)
Glass (8%)
Garden Waste (6%)
Other (3%)

The Martin's family garbage weighs 38 kg.

Use the diagram above to calculate the amount of food refuse that is in the tin.

Food $0.49 \times 38 = 18.62$ kg

Paper $0.25 \times 38 = 9.5$ kg

Glass $0.08 \times 38 = 3.04$ kg

Plastic $0.09 \times 38 = 3.42$ kg

Garden Waste

$0.06 \times 38 = 2.28$ kg

Other $0.03 \times 38 = 1.14$ kg

Quadratic Trinomials.
A quadratic expression has the form $ax^2 + bx + c$, where $a \neq 0$. Sometimes there will be no "bx" part and sometimes no "c" part (if $b = 0$ or $c = 0$). A quadratic trinomial has all 3 parts - the x^2 part, the bx part and a number, c. Sometimes you will be given a quadratic trinomial and be asked to factorise it. This means that you have to find an expression with two sets of brackets that expand into the quadratic.

CREDIT CRUNCH

Alicia receives an offer for a credit card in the mail. The bank offers "A Great Deal".
If she signs up for their credit card they will give her a $500 limit with minimum monthly payments of $20 and interest of 21%

The offer sounds too good to be true so Alicia signs up and immediately goes out and purchases a new cell phone for $500.

1. Estimate how long will it take Alicia to pay off the $500 by making the minimum $20 payment each month. You may wish to use a spreadsheet with headings similar to below.

Statement	Amount Due	Payment	Balance	Interest 21% ÷ 12 = 1.75%	Balance Due
1	$500	$20	$480	$8.40	$488.40
2	$488.40	$20	$468.40	$8.20	$476.60
3	$476.40				

2. What will be the total amount paid to the credit card company and what will be the actual interest paid?

It will take 33 months (payments) of $20 payments.

The total amount paid will be 33 × $20 = $660

This means interest will be $660 ÷ $500 = 1.32 or 32% interest

A SOUND INVESTMENT?

1. The value of a car decreases over time. Suppose you purchase a car for $10 000. The cars value t years after purchase can be approximated by the function: $V = 10\,000(0.8)^t$. Complete the table below and find out after how many years the car will be worth less than $5000.

Year (t)	0	1	2	3	4	5
Car Value $V = 10\,000(0.8)^t$	$10 000	8000	6400	5120	4096	3277

Between years 3 and 4 the car will be worth less than $5000

BLACK Worksheet 23

FACTORS, SUMS and AREAS

1. Each of the top numbers in the table below can be factored as a product of two numbers whose sum is the bottom number in the column. Fill in all the missing numbers.

Product	36	56	63	72	96	90	180	63	168	432
Factor	18	8	9	36	12	9	15	21	8	24
Factor	2	7	7	2	8	10	12	3	21	18
Sum	20	15	16	38	20	19	27	24	29	42

2. Write the missing products and dimensions of each quadrilateral.
   Write an expression that gives the area of the figure.
   The figures are not drawn to scale.

x^2	6x
2x	12

x

2

x

6

Area = $x^2 + 8x + 12$

x^2	5x
3x	15

x

3

x

5
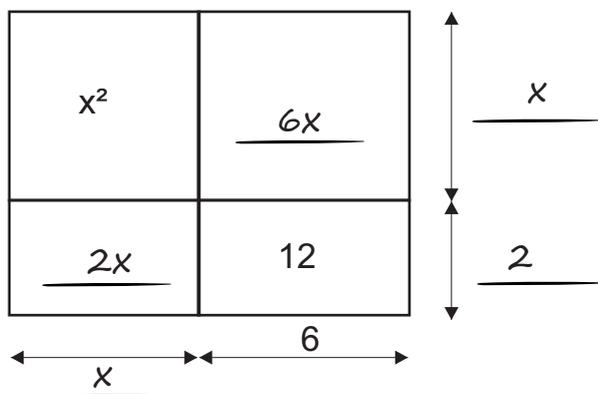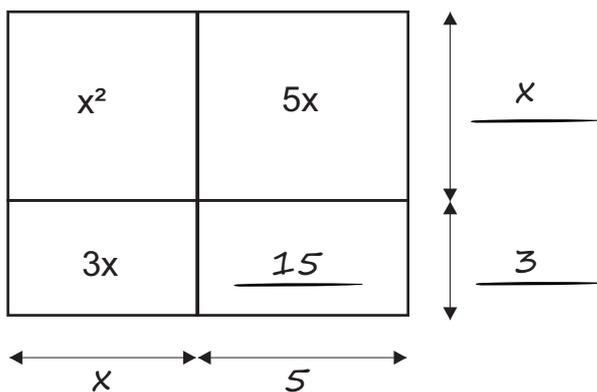
Area = $x^2 + 8x + 15$

CONSTRUCTIONS

You are going to construct a triangle similar to the one below.
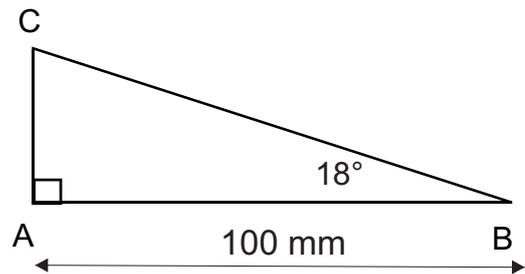
1.   Construct a line through A, perpendicular to AB.

2.   Use your protractor to draw ABC = 18º and complete the triangle ABC.

3.   Using the diagram you have drawn, measure the length of BC to the nearest millimetre.

105 mm
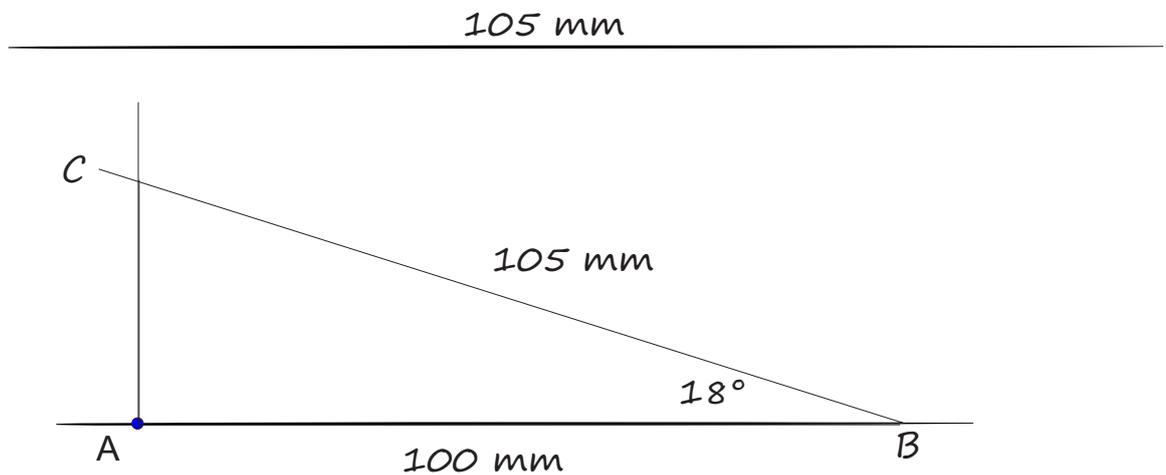
4.   Use Trigonometry to verify your BC length answer.
$$Cos\ 18° = \frac{100}{CB}$$

      $100 ÷ Cos\ 18° = 105.15\ mm$

5.   Measure the length of AC then use either Trigonometry or Pythagorus to verify your answer. Explain one reason why the answers could be different.

      Measuring = 32 mm. ALLOW ± 1 or 2 mm accuracy.

      Using Pythagorus AC = $\sqrt{105^2 - 100^2}$

      = 32.0 mm

      Using Trig, Sin 18° × 105 = 32.44 mm

      or Tan 18° × 100 = 32.49 mm

      Accuracy of the answers depends on the hypotenuse CB measure.

GRAPHING AREAS

The diagram shows three rectangles.
Their base and height measurements add up to 9 cm.

7cm

2cm

6cm

3cm

4cm

5cm
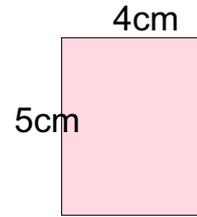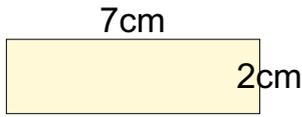
The table below shows the area of rectangles with base and height adding to 9 cm.

1. Complete the table then use the table reults to draw complete the graph below.

Base of Rectangle	Height of Rectangle	Area of Rectangle
8 cm	1 cm	8 cm²
7 cm	2 cm	14 cm²
6 cm	3 cm	18 cm²
5 cm	4 cm	20 cm²
4 cm	5 cm	20 cm²
3 cm	6 cm	18 cm²
2 cm	7 cm	14 cm²
1 cm	8 cm	8 cm²

Graph Comparing
Rectangle Heights v Area

(4.5, 22.25)

Notice the open points
if base was 0 cm or 9 cm
the rectangle would not
be valid

Area of rectangle (cm²)

Base length (cm)
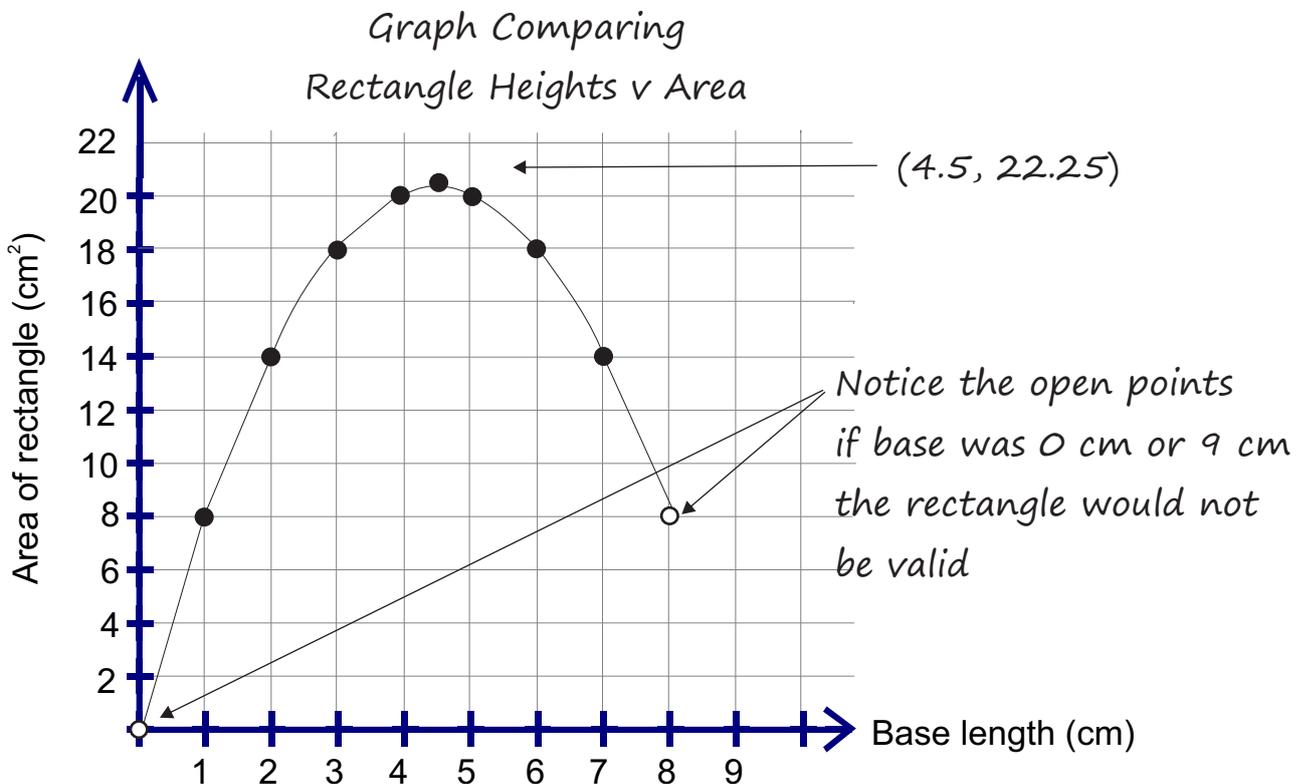
2. What is the greatest area possible for a rectangle if its base and height add up to

If base 4.5 cm, height = 4.5 cm, Area = 22.25 cm²

CIRCULAR MATHEMATICS

Stepping the radius around the circumference of a circle, gives six equal parts. If these points are then joined to the centre of the circle, 6 triangles are formed.

1.   What sort of triangles are formed from this process?
Give a reason for your answer.

Equilateral triangle. All sides are the same length

i.e. all are the size of the radius.

2.   What is the size of each angle in the triangles?

60°

3.   If the radius of the circle is 5 cm calculate the area of the triangle that is shaded.

Vertical height is 4.33 worked out using Pythagoras

height $=\sqrt{5^2 - 2.5^2}$ *or by using trig Tan 60° × 2.5 cm*

Therefore Area is ½ × 5 × 4.33 = 10.8 cm² (1 dp)
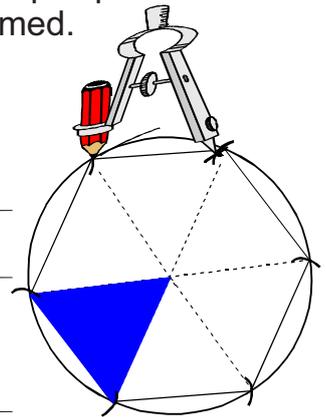
Edith decides to make a skirt. The pattern's shape forms part of a sector of a circle.

4.   What fraction of a circle is this sector?

Fraction is: $\dfrac{120}{360} = \dfrac{1}{3}$

35 cm

120°

57 cm
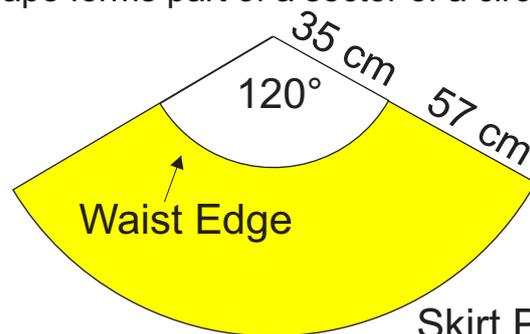
Waist Edge

Skirt Pattern

5.   The circumference of a circle is measured by the formula ___*C = 2πr*___

6.   Use your formula to calculate the length of the waist edge on the pattern to the nearest whole number.

C = 2πr, C = 2 × 3.14 × 35 × $\dfrac{1}{3}$

= 73.26 cm,

To the nearest cm, Length of Waist Edge = 73cm

7.   How many square metres of material is being used in the dress?

Remember to convert to metres.

A = πr², $\dfrac{1}{3}$*(3.14 × 0.92²) –* $\dfrac{1}{3}$*(3.14 × 0.35²)*

= 0.76 m²

BLACK Worksheet 24
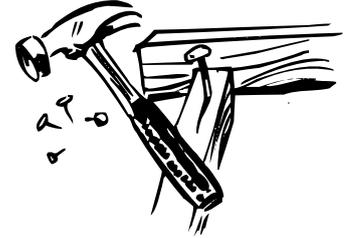
MATHEMATICS IN TECHNOLOGY

Forces and lever arm distances obey an "inverse proportion".

Applied Force (F1)

Lift Force (F2)

$$\frac{F1}{F2} = \frac{L2}{L1}$$

L1

L2
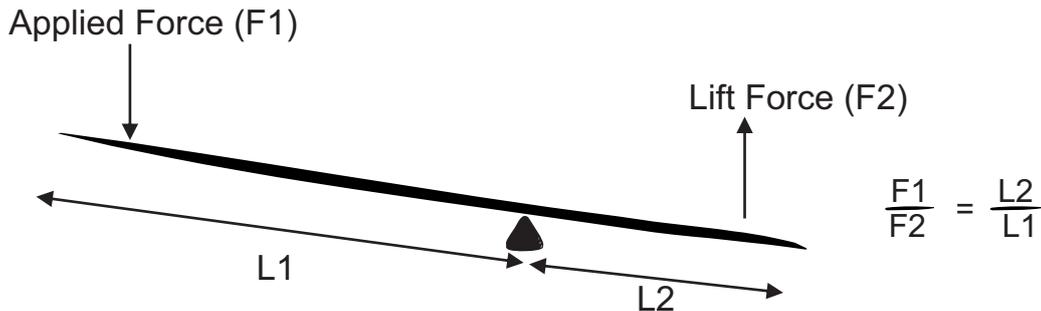
1. A 100 kg weight is applied to one end of a leaver 2.2 metres long.
The leaver is pivoted 0.2 metres from its end. What lift force is exerted?

$$\frac{F1}{F2} = \frac{L2}{L1} \qquad\qquad \frac{100}{F2} = \frac{0.2}{2.0}$$

Remember the lever is 2.2 metres long, L1 = 2.0 m, L2 = 0.2 m

$100 \times 2 = F2 \times 0.2$

$200 \div 0.2 = F2$

$F2 = 1000 \ kg$

(we measure force in Newtons)   = 10 000 Newtons

2. Pulleys transfer power in much the same way as a gear. Different speeds can be obtained by changing the sizes of the pulley diameters and the speed of the driving pulley. For the pulley system shown below, the speed of the pulley is inversely proportional to its diameter. If Pulley A has a diameter of twice that of Pulley B then when Pulley A makes one turn, Pulley B will make 2 turns (assuming there is no slippage in the belt).

$$\frac{\text{Speed of Pulley A}}{\text{Speed of Pulley B}} = \frac{\text{Diameter of Pulley B}}{\text{Diameter of Pulley A}}$$

Pulley A   Pulley B

If Pulley B is 10 cm and is rotating at 240 rpm (revolutions per minute) then calculate the speed of Pulley A if its diameter is 25 cm.

$$\frac{A}{240} = \frac{10}{25} \qquad 25A = 240 \times 10$$

$25A = 2400$

Speed Pulley A = 96 rpm (2½ times slower)

MATHEMATICS AND PACKAGING APPLICATIONS

Evergo batteries are shaped and packed as in the
diagram. Batteries are 2 cm in diameter and 5 cm long.
Batteries are packed in rectangular boxes which are 16
cm long and 10 cm wide.
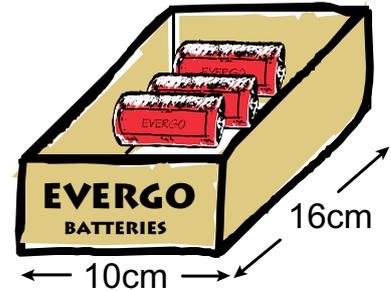
1.   If batteries are packed as in the diagram, how many will fit on the bottom layer of
      the cardboard box?

         Number of batteries = 16

2.   The batteries are packed in two layers, as in the diagram below:
      The batteries are sold for 75 cents each.
      What is the cost of a box of batteries?
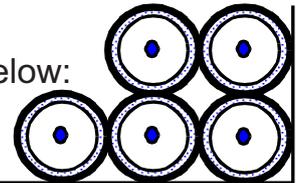
         Number of batteries per box = 32

         ∴ 32 × 0.75 = $24

3.   Some shops give a 10% discount if customers purchase a box of batteries.
      How much would a box of batteries cost if a 10% discount was given?

         Discount = $2.40

         Cost with discount = $21.60

MATHEMATICS AND SCIENCE APPLICATIONS

1.   Adele has 20 milliliters of a 30% solution of Sulphuric Acid. She needs a 25%
      solution for a class experiment. She also has a bottle of 15% solution of Sulphuric
      Acid. She needs to calculate how many milliliters of the 15% solution she should
      add to obtain the required 25% solution. She draws up the following table:

	Amount of Solution (mL)	Amount of Sulphuric Acid
30% solution	20	0.30(20)
15% solution	x	0.15x
25% solution	20 + x	0.25(20 + x)

      She then forms an equation to solve: $0.30(20) + 0.15x = 0.25(20 + x)$.
      Solve the equation and find how much of the 20% solution should be added.

         6 + 0.15x = 5 + 0.25x

         1   = 0.1x

         x   = 10 mL

         10 mL of the 25% solution should be added.

CONSTRUCTIONS

1. Draw an accurate pattern of the shape shown.
You need to:
i. Use a compass and ruler to find point B.
ii. Construct a perpendicular bisector of AB.
iii. Draw the semicircle with AB as diameter.
iv. Find point C (Length AC = 60 mm, Length BC = 85 mm).
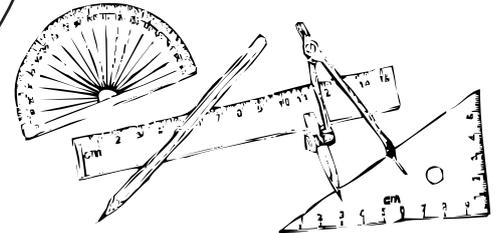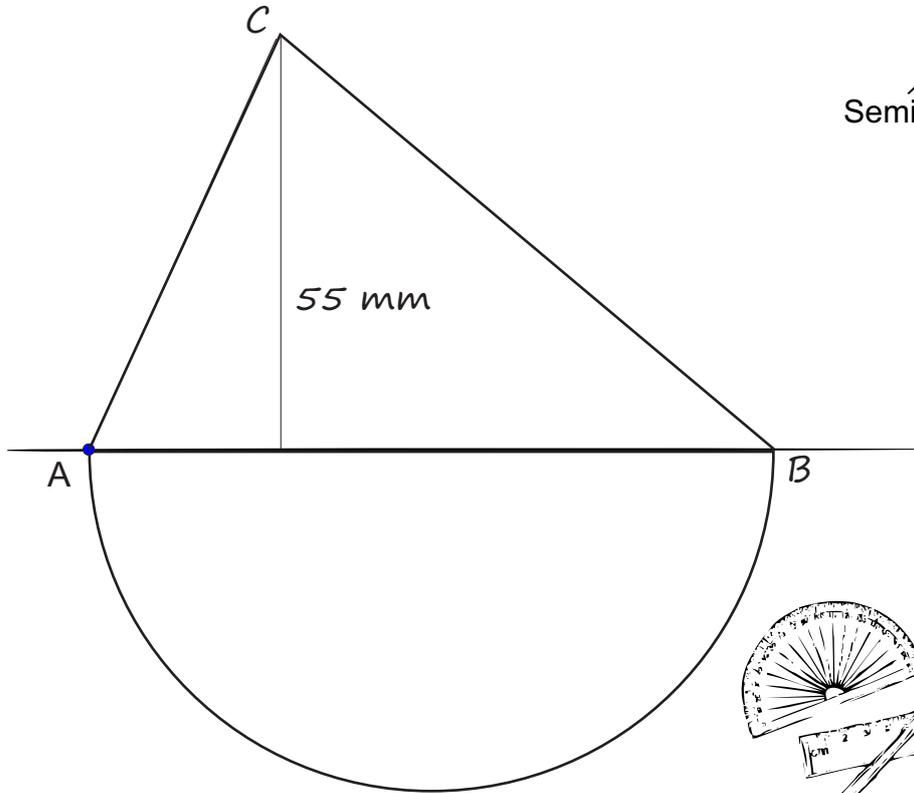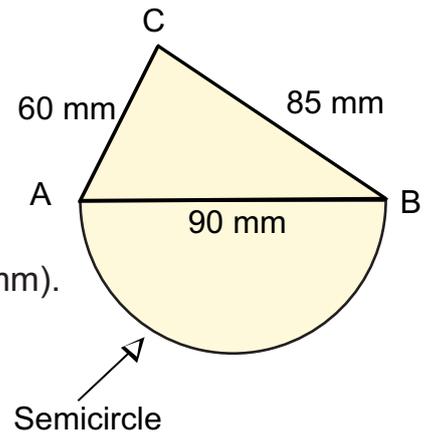
Semicircle

2. Calculate the area of the shape you have drawn.

Measuring the vertical height of the triangle = 55mm.

therefore the area = ½ × 90 × 55

   = 2475 mm²

Area of semi-circle is ½ × ¶ × 45² = 3181 mm²

Total Area – add them together 2475 + 3181

Total Area = 5656 mm²

(the actual measurements using trigonometry are

vertical height = 54.67 mm, Area = 2460 mm²)

Did you know that you can check your answer by using the online calculators at the mathscentre website? Just use the triangle and the sector calculators to work out each separate area. The add together the two areas

ALGEBRA

1. Expand and simplify each expression

$3(x + 2) + 2(x + 3)$

$3x + 6 + 2x + 6$

$= 5x + 12$

$4(x + 1) + 5(x + 1)$

$4x + 4 + 5x + 5$

$= 9x + 9$

$7(x + 2) + 2(x - 2)$

$7x + 14 + 2x - 4$

$= 9x + 10$

$5(x - 4) + 3(x - 2)$

$5x - 20 + 3x - 6$

$= 8x - 26$

$7(x + 2) - 2(x + 2)$

$7x + 14 - 2x - 4$

$= 5x + 10$

$5(x + 4) - 3(x - 2)$

$5x + 20 - 3x + 6$

$= 2x + 26$

$4(x - 3) - 2(x + 2)$

$4x - 12 - 2x - 4$

$= 2x - 16$

$3(x - 6) - 4(x - 3)$

$3x - 18 - 4x + 12$

$= -x - 6$

$5(x - 2) - 2(x + 1)$

$5x - 10 - 2x - 2$

$= 3x - 12$

2. Factorise the following expressions by completing.

$12x + 6 = 6(\underline{\quad 2x + 1 \quad})$

$16x - 12 = 4(\underline{\quad 4x - 3 \quad})$

$6x + 18 = \underline{\quad 6 (x + 3 \quad)}$

$24 - 20x = \underline{\quad 4 (6 - 5x)}$

$12x - 9 = \underline{\quad 3 (4x - 3 \quad)}$

$2x + 4y = 2(\underline{\quad x + 2y \quad})$

3. Factorise the following:

$10 - 2a = \underline{\quad 2(5 - a) \quad}$

$12b - 9 = \underline{\quad 3(4b - 3) \quad}$

$8g + 20 = \underline{\quad 4(2g + 5) \quad}$

$16y + 12 = \underline{\quad 4(4y + 3) \quad}$

$6x - 20 = \underline{\quad 2(3x - 10) \quad}$

$9 - 24d = \underline{\quad 3(3 - 8d) \quad}$

$x^2 + 7x = \underline{\quad x(x + 7) \quad}$

$a^2 - a = \underline{\quad a(a - 1) \quad}$

$a^2 p + a^2 = \underline{\quad a^2(p + 1) \quad}$

$10b^2 - b = \underline{\quad b(10b - 1) \quad}$

4. Factorise the following:

$2x + 2y + ax + ay = \underline{\quad 2(x + y) + a(x + y) \quad}$

$= \underline{\quad (2 + a) (x + y) \quad}$

$3x + 12 + xy + 4y = \underline{\quad 3(x + 4) + y(x + 4) \quad}$

$= \underline{\quad (3 + y) (x + 4) \quad}$

BLACK Worksheet 25

PERCENTAGES FOR TRADESMAN

1. A mechanic buys an air filter for $47.98. His mark up is 140%. What does he charge a customer for the airfilter?

Markup: $47.98 × 1.4 = $67.17
Price:$47.98 + $67.17 = $115.15

2. The electricity line voltage is 225 volts. During a system malfunction the voltage drops 5.5%. What is the reduced voltage?

225 × 0.945 = 212.6 volts

3. Due to a change in specifications the cost of a new kitchen increases by 25%. If the original cost was $16 750, what will be the new cost?

$16750 × 1.25
= $20937.50

4. By using an automatic welding machine the time taken to weld a new tank can be reduced by 40%. Originally it was budgeted to take 30 hours. How long will it take with the machine?

30 hours × 0.6
= 18 hours

5. Air-conditioning engineers reduce a heating duct from 1.1 metres to 0.96 metres. What was the percentage decrease in length?

Amount of decrease = 0.14m
$\frac{0.14}{1.1}$ *= 12.7%*

6. The reading on a pressure valve increases from 35 psi to 42 psi. What is the percentage increase?

Amount of increase = 7 psi
$\frac{7}{35}$ *= 20%*

7. A car's water pump costs $82.20. The pump is sold to a customer for $205.50. What is the percentage markup?

Markup = $123.30
$\frac{123.30}{82.20}$ *= 150%*

THE RIGHT DIMENSION

The diagram represents a grain hopper. The hopper has a length of 4x metres and has a constant cross-sectional area comprising of a square of side length x metres and a triangle with a perpendicular height of ½ x metres.
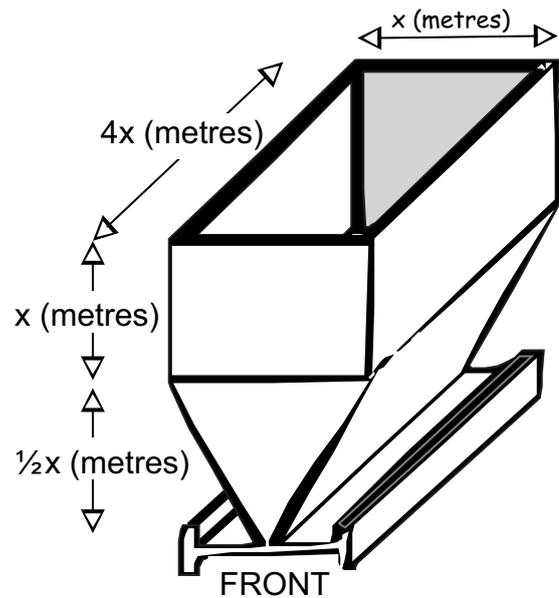
1. Find in terms of x, the area of the cross-section (of the front) of the hopper.

Cross sectional area
= Area Square + Area triangle

$$= x^2 + \frac{1}{2} \times x \times \frac{1}{2} x$$

$$= x^2 + \frac{1}{4} x^2$$

$$= \frac{5x^2}{4} \ m^2$$

2. What is the volume of the hopper?

Volume = Cross sectional Area × Length: $V = \frac{5x^2}{4} \times 4x$

$$= 5x^3 \ (m^3)$$

3. The hopper can hold 40 m³ of grain.
Calculate the size of x.

$5x^3 = 40$

$x^3 = 8$, i.e size of $x = 2$ m

4. Find the area of the arrow shown below.

←1.5y (cm)→

2y (cm)

3y (cm)
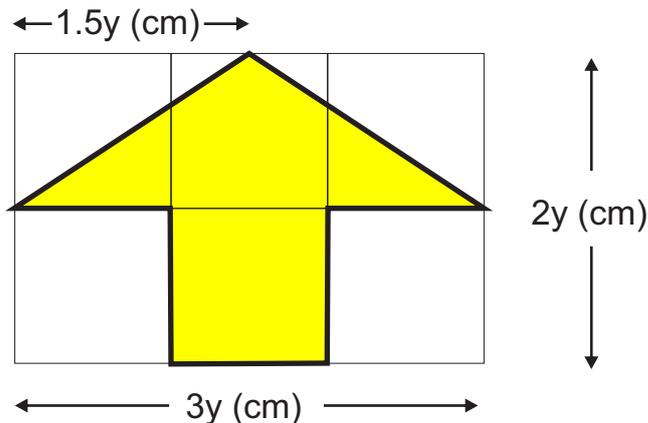
Area of bottom square

$= y^2 \ (cm^2)$

Area of top triangle

$= ½ \ 3y \times y \ (½ \times base \times height)$

$= 1.5y^2 \ (cm^2)$

Total Area

$y^2 + 1.5y^2 = 2.5y^2 \ (cm^2)$

5. If 2x + y = 10 cm, what is the value of 4x?

$2x = 10 \ cm - y$

$4x = 20 \ cm - 2y$

LINEAR GRAPHS AND REAL LIFE APPLICATIONS - DIVING

Quite often, diving books work in imperial measurements. This means that depths are given in feet and inches and pressure in terms of psi (pounds per square inch). Most diving manuals state that underwater pressure increases by 4.3 pounds per square inch (psi) for every 10 feet you descend.

This can be expressed by the equation p = 0.43d + 14.7

    p = the pressure in pounds per square inch
    d = the depth of the dive in feet

  i.   Draw a graph of this equation.
  ii.   Use your graph to estimate the depth where you would reach 150 psi.

Graph of Dive Depth v Pressure
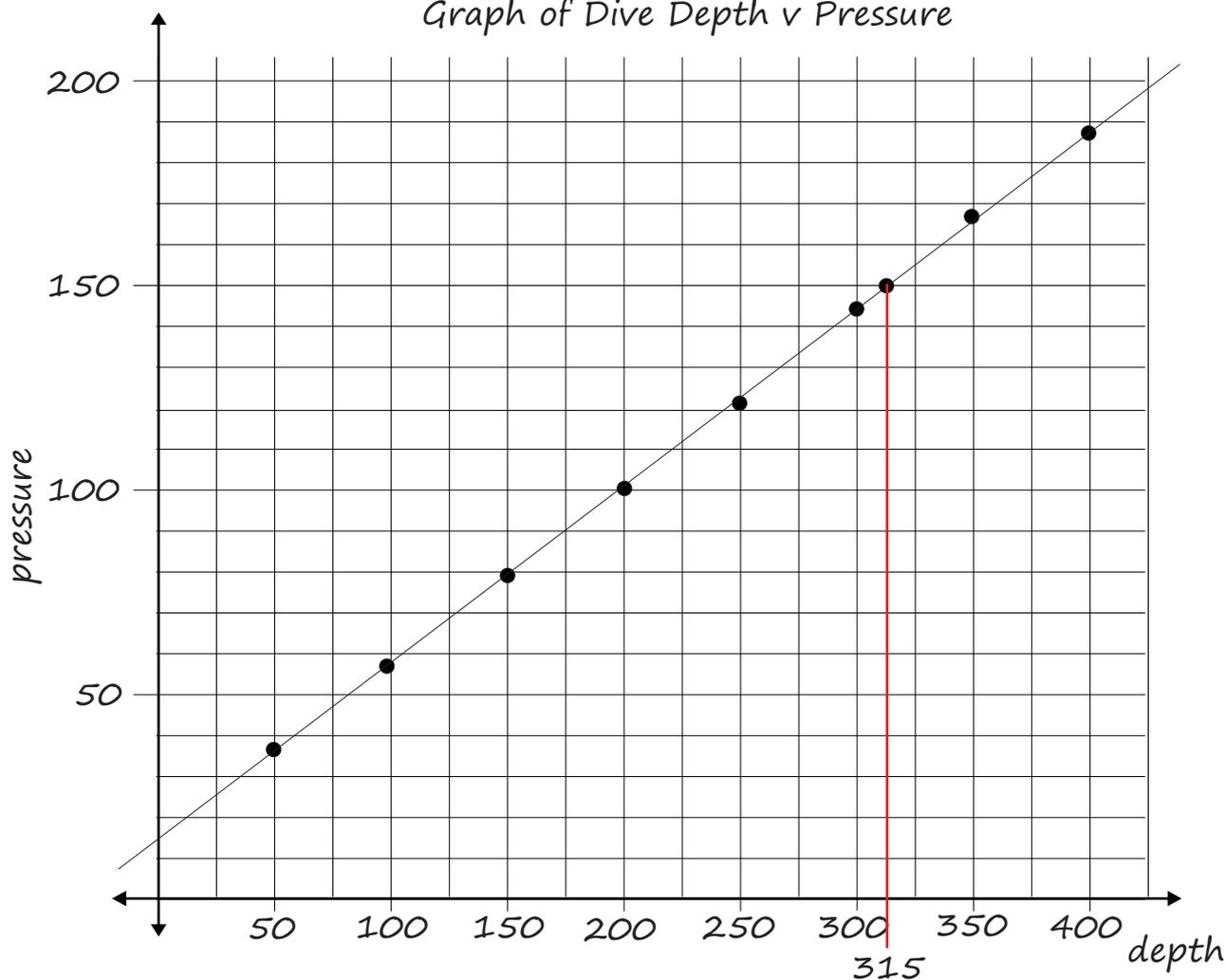
DIVE DEPTH	50	100	150	200	250	300	350	400
PRESSURE p=0.43d + 14.7	36	58	79	101	122	144	165	187

At depths around 315 feet, pressure is estimated at 150 psi.

ALGEBRA WORKOUT

Expand and simplify:

$-5(x - 5y)$

$-5x + 25y$

$5x + 2(x + 6)$

$5x + 2x + 12$

$7x + 12$

$(x + 6)(x + 5)$

$x^2 + 5x + 6x + 30$

$= x^2 + 11x + 30$

$(x + 4)(x - 4)$

$x^2 + 4x - 4x - 16$

$= x^2 - 16$

$(x + 3)(2x - 5)$

$2x^2 - 5x + 6x - 15$

$= 2x^2 + x - 15$

$(x - 7)(x - 6)$

$x^2 - 6x - 7x + 42$

$= x^2 - 13x + 42$

$(x - 10)^2$
$(x - 10)(x - 10)$

$= x^2 - 10x - 10x + 100$

$= x^2 - 20x + 100$

$\dfrac{x^2 - 9}{x^2 - 4x + 3}$

$\dfrac{(x - 3)(x + 3)}{(x - 3)(x - 1)}$

$\dfrac{(x + 3)}{(x - 1)}$

$(2x + 7)(3x - 6)$

$6x^2 - 12x + 21x - 42$

$= 6x^2 + 9x - 42$

Factorise:

$3a^2 - ab$
$a(3a - b)$

$4x^2 - 16$
$(2x - 4)(2x + 4)$

$1 - x^2$
$(1 - x)(1 + x)$

$3x^2 - 12$
$3(x^2 - 4)$
$= 3(x + 2)(x - 2)$

$x^2 - 4x + 4$
$(x - 2)(x - 2)$

$x^2 + 6x - 16$
$(x + 8)(x - 2)$

$x^2 - 5x - 6$
$(x - 6)(x + 1)$

$x^2 + 12x + 36$
$(x + 6)(x + 6)$

$2x^2 + 14x + 20$
$2(x^2 + 7x + 10)$
$= 2(x + 5)(x + 2)$

Solve:

$40x = 10$

$x = 0.25$

$x^2 = 4x + 60$

$x^2 - 4x - 60 = 0$

$(x - 10)(x + 6) = 0$

$x = 10, x = -6$

$\sqrt{16x^2} = 20$

$16x^2 = 400$

$x^2 = 25$

$x = 5$

BLACK Worksheet 26

MATHEMATICS AND NUTRITION

Most foods are required by law to show factual information regarding nutrition. Below is the information taken from a label on Puffed Wheat Breakfast Cereal. The '% Daily Intake' figures are based on a 2000 calorie diet. For breakfast, you like to eat 2 cups of Puffed Wheat with 1 cup of skim milk.

Nutrition Facts

Serving Size: 1 cup (30 grams)
Servings per container: 18

Amount per serving	Puffed Wheat	With ½ cup skim milk
Calories	120	160
Calories from fat	10	10
	% Daily Intake	
Total Fat	1%	2%
Cholesterol 0 mg	0%	1%
Sodium 220 mg	9%	12%
Potassium	3%	9%
Total Carbohydrates 24g	8%	10%
Dietary Fibre 3g	12%	12%
Sugars 4g		
Vitamin A	10%	15%
Vitamin C	10%	10%
Calcium	0%	15%
Iron	45%	45%
Vitamin D	10%	25%
Thiamin	50%	50%
Riboflavin	50%	60%
Vitamin B6	50%	50%
Folic Acid	50%	50%
Phosphorus	10%	20%
Magnesium	8%	10%
Zinc	50%	50%
Copper	4%	4%

1. How many calories are consumed with your breakfast cereal?

   160 × 2 = 320 calories

2. What percent of the daily intake of fat do you consume?

   1 cup is 2% therefore your
   2 cups = 4% of daily fat

3. You increase the milk on your cereal to 1½ cups. How many extra calories will you consume?

   You are increasing by 40 calories.

4. Nutritionalists recommend that you eat a breakfast food that provides at least 10% of daily Vitamin C intake. Do you get this with your bowl of Puffed Wheat?

   Yes you get 10% with 1 cup
   You are getting 20% (2 cups)

5. Nutritionalists recommend that 20% of your daily calories come from breakfast. You are told that you need at 2500 calories during the day. Are you getting the right amount of calories with your breakfast of Puffed Wheat with milk?

   20% of 2500 calories means breakfast should contain 500 calories.
   You are only getting 320 calories so more than 1 bowl of Puffed Wheat is needed.

USING MATHEMATICS AT THE POOL

1. A rectangular shaped swimming pool is used at Mahobe College for interschool swimming competitions. It is 50 metres long, 32 metres wide and 2.4 metres deep. Once a year the pool is drained for service. There are two drains, one at each end of the pool. Drain 1 empties the pool at 4.5 cubic metres of water per minute. Drain 2 empties the pool at a rate of 5.5 cubic metres of water per minute. If both drains are opened at 9 am, when will the pool be empty?

Volume of pool is 50 × 32 × 2.4 = 3840 m³

Drain 1: Rate is 4.5 × 60 = 270 m³ per hour

Drain 2: Rate is 5.5 × 60 = 330 m³ per hour

Therefore the total rate of drain each hour is 600 m³

3840 m³ ÷ 600 m³ per hour = 6.4 hours (or 6 hours 24 minutes)

Therefore 9 am + 6 hours 24 minutes = 3.24 pm

2. After servicing, the caretaker fills the pool using 3 hoses.
Hose 1 fills at a rate of 2.25 m³ of water per minute.
Hose 2 fills at a rate of 1.5m³ of water per minute.
Hose 3 fills at a rate of 2 m³ of water per minute.

If the hoses are turned on at 6 am, will the pool be ready for afternoon swimming lessons?

Volume of pool = 3840 m³

Hose 1: Rate is 2.25 × 60 = 135 m³ per hour

Hose 2: Rate is 1.5 × 60 = 90 m³ per hour

Hose 3: Rate is 2 × 60 = 120 m³ per hour

Therefore the total fill rate each hour is 345 m³

3840 m³ ÷ 345 m³ per hour = 11.13 hours (or 11 hours 8 minutes)

Therefore 6 am + 11 hours 8 minutes = 5.08 pm

The pool would not be ready for afternoon swimming.

3. Sports scientists have calculated that swimmers burn approximately 10.5 calories per minute. The equation C = 10.5t represents C (the number of calories burnt) and t (the time in minutes spent swimming). If one particular swimmer wanted to burn 400 calories, for how long should they swim?

400 = 10.5 t

400 ÷ 10.5 = t

t = 38.10 minutes

They should swim at for at least 38 minutes

MATHEMATICAL GRAPHS
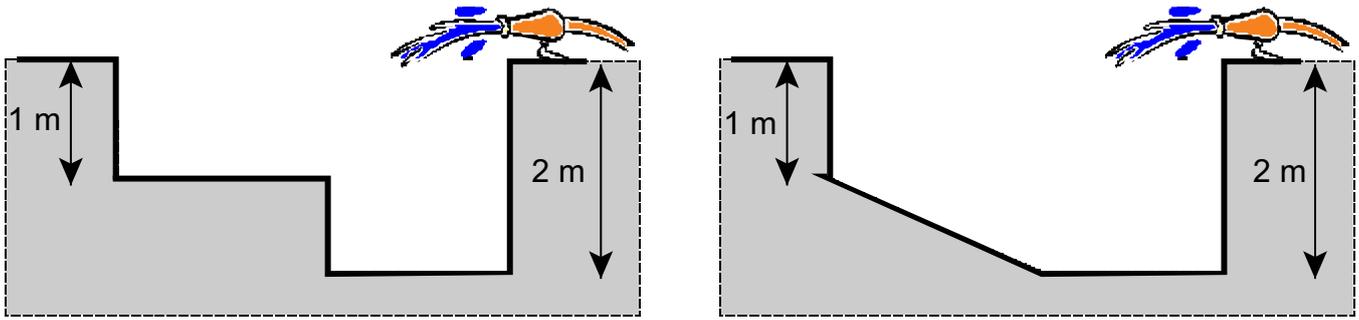
Two swimming pools are filled using a hose which pours water at a constant rate.
The cross sections of both pools are drawn below.

1.   Describe in words how the depth of the water in the deep end of each pool varies
   with time from the moment that the pools first begin to fill.

      Pool A: *Fills at a constant rate for the first 1 metre. It then fills at*

      a constant but slower rate.

      Pool B: *For the first 1 metre, the pool fills at an increasingly slower*

      rate (because it is filling more and more volume).

      After 1 metre it fills at the same rate as Pool A.

      (i.e after 1 metre the graph lines are drawn with same gradient.)

2.   Sketch a graph showing how the depth of water in the deep end varies with time.
      Clearly label your graph Pool A and Pool B.

Filling the Swimming Pool

Depth of Water

Time (minutes)

——— Pool A   · · · · · · Pool B
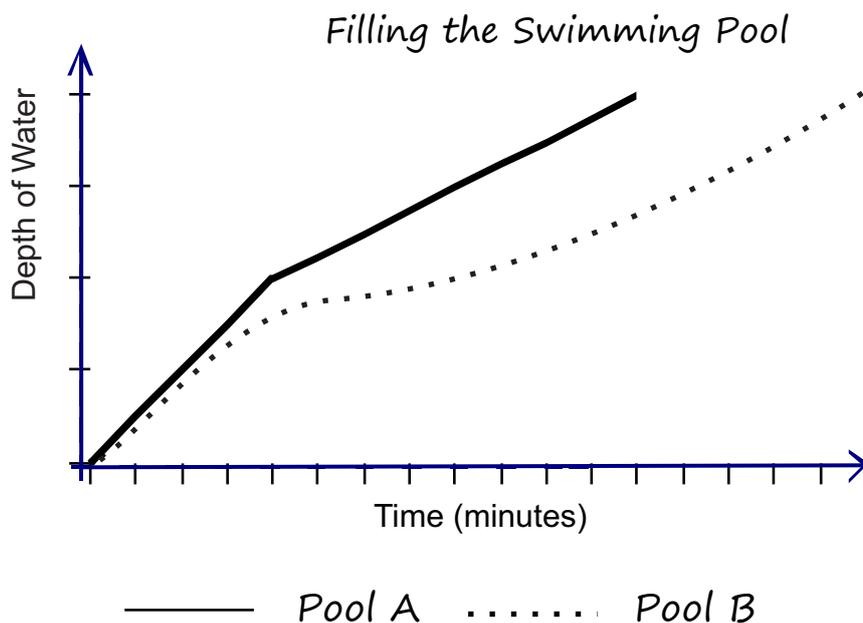
LINEAR GRAPHS AND HUMAN BIOLOGY

The amount of blood in a body can be approximately calculated by using the equation:

$$A = \frac{w}{14}$$

A = the amount of blood in litres
w = the weight of the person in kilograms.

Graph the equation and predict the weight of a person who has 6.5 litres of blood.

Graph of Weight v Blood Volme
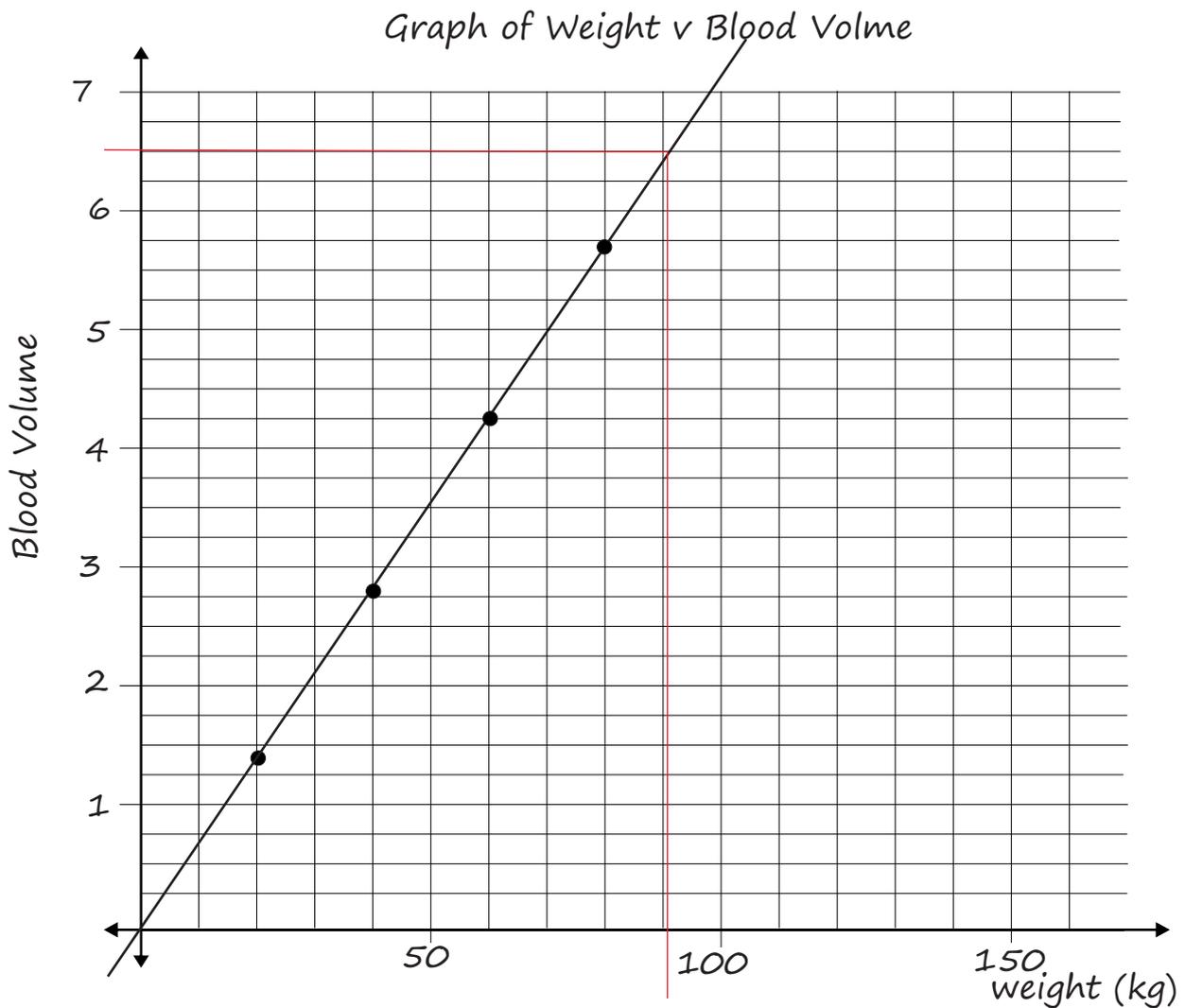
WEIGHT (KG)	20	40	60	80	100
BLOOD VOLUME $\frac{w}{14}$	1.4	2.9	4.3	5.7	7.1

At 91 kg, blood volume will be 6.5 litres.

BLACK Worksheet 27

SOLVING APPLIED PROBLEMS

Then number of diagonals (d) of a polygon with n sides is given by the formula below:

$$d = \frac{n^2 - 3n}{2}$$

1. Use the formula to find how many diagonals an octagon has.

$d = \frac{8^2 - 3 \times 8}{2}$ (An octagon has 8 sides.)

$d = \frac{64 - 24}{2}$ $d = 20$ diagonals

2. When the adult dosage is known, nurses use "Young's Rule" to determine the amount of medicine to give a child under the age of 12.

$$C = \frac{AD}{A + 12}$$

C = Child's dose
A = Age of child
D = Dose normally given to an adult

Amoxicillin is a powerful antibiotic given for serious infections. The recommended adult dosage is 250 mg every 8 hours. Using Young's Rule, what would be the recommended dosage for a 12 year old child?

$\frac{12 \times 250}{12 + 12} = \frac{3000}{24}$

$= 125$ mg every 8 hours

3. When several electrical appliances are used at home the total current being drawn is the sum of all the individual currents drawn by each appliance. If two appliances W1 and W2 are being used at the same time, the current flow, I (amps), can be expressed by the formula below. Note - New Zealand voltage (V) is a constant 230 volts.

$$I = \frac{W1}{V} + \frac{W2}{V}$$

W = Power in watts
V = Voltage (V)

Sally is drying her hair with a hair dryer that is rated at 1600 watts. Her TV is also playing in the room. What power would the TV be rated at if the total current flow is 7.2 amps?

$\frac{1600}{230} + \frac{W2}{230} = 7.2$ amps

$\frac{W2}{230} = 7.2 - 6.96$

$W2 = 0.24 \times 230$

$W2 = 55.2$ watts

MORE MATHEMATICS

1. A Forensic Anthropologist can determine age, sex and stature of a body by examining teeth and bones. When remains of ancient people have been found, anthropologists can determine their height by using the formula H = 82 + 2.5T (for males) or H = 73 + 2.5T (for females). H represents the height of the person (in cm) and T is the length of the body's tibia bone found.
Complete the table, then graph the resultant pairs of numbers.

MALE		FEMALE	
tibia length (cm)	Height	tibia length (cm)	Height
30	157	30	148
34	167	34	158
38	177	38	168
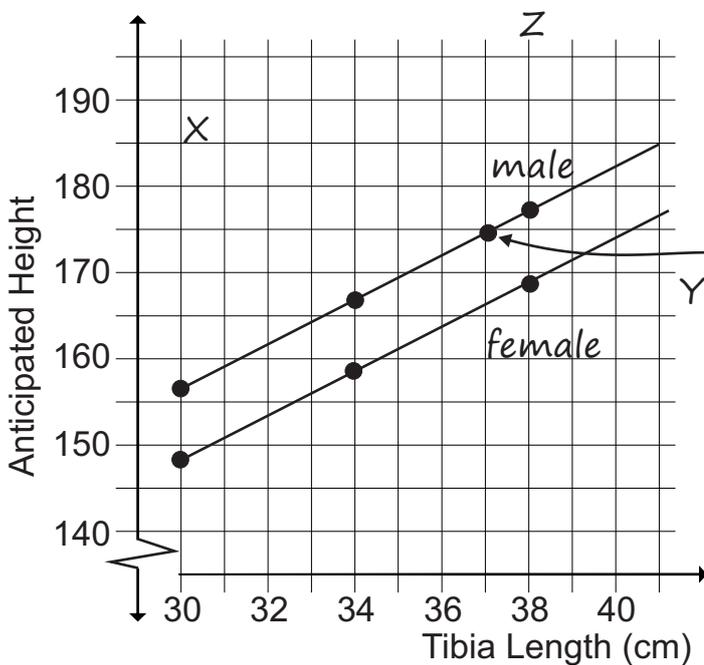
2. If bones are found and you are told that the original body was a 1.74 m male. How long (approximately) should the tibia bone be?

Using the graph – approx

37 cm.

3. An animator draws an arrow head on a grid. She then move the arrow head 4 units right, 2 units up and reflects it in the line ab.

Draw the resulting image
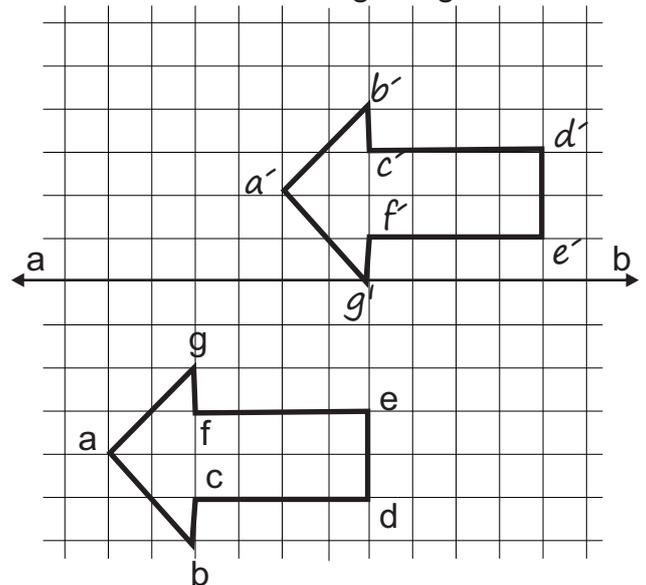
4. Reflect the octagon over each of its sides.
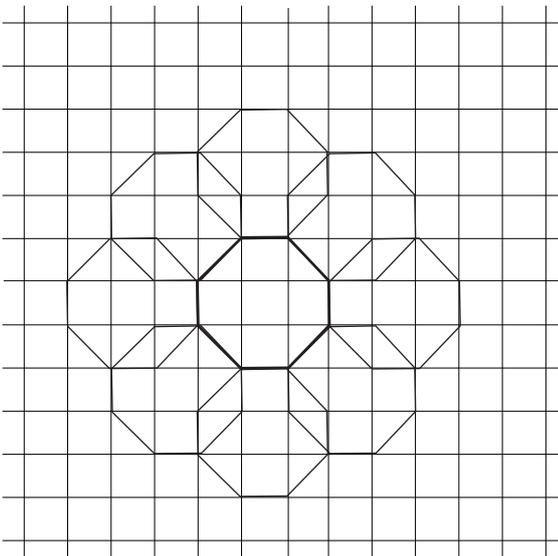
LINEAR LINE GRAPHS
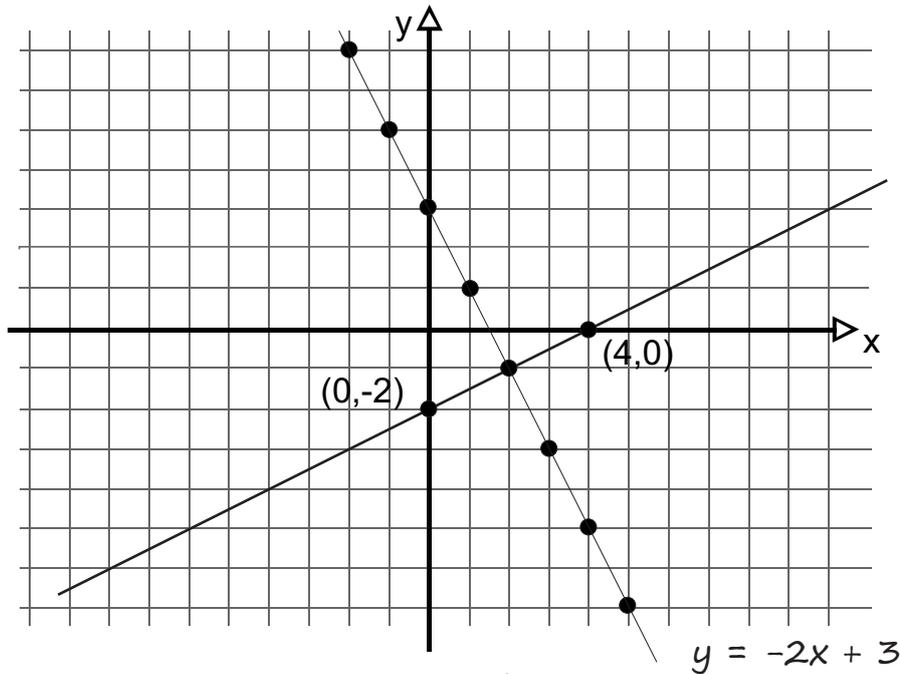
$y = -2x + 3$

The diagram above shows the graph of $y = \frac{1}{2}x - 2$.

1. What is the gradient of this line? $\frac{1}{2}$

2. On the diagram, draw the graph of $y = -2x + 3$.

3. Using the graph, estimate the point of intersection of the lines: $y = \frac{1}{2}x - 2$ and $y = -2x + 3$.

$(2, -1)$

4. Write an equation of any line that is parallel to $y = \frac{1}{2}x - 3$.

Equation must have a gradient of ½ e.g. $y = ½x + 2$, $y = ½x$, $y = ½x - 7$

5. The line $y = -2x + 3$ is reflected in the y axis.
What is the equation of the new line?

Equation of reflected line $y = 2x + 3$

6. In the figure shown, choose any line which has:

a positive gradient

Positive gradient: a and c.

a negative Y intercept

Negative intercept: c and d
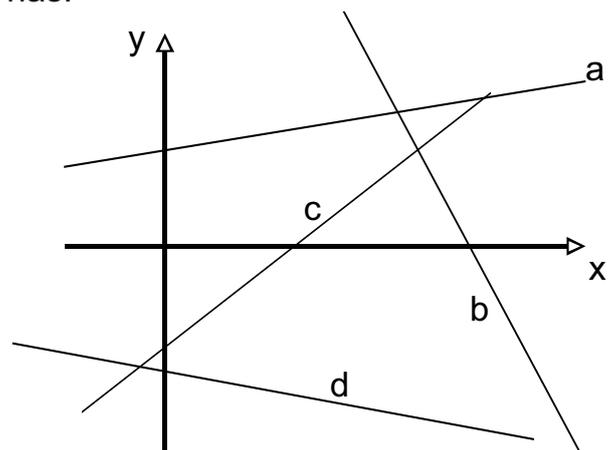
PAINTING BY NUMBERS

You are are given the task of painting the walls and door of a barn.
The diagram shows the barn. The front and back walls are the same shape.
The two sides are the same rectangular shape and each have 3 windows with dimensions 2 m × 1 m.

1.   Calculate the total area to paint.

3.5 m

2.1 m

10.4 m   20 m

Area:2 Side Walls,   2 x 2.1 x 20 = 84

2 End Walls, 2 x 10.4 x 2.1 = 43.68

+   2 x ½ x 10.4 x 1.4 = 14.56

Total Area (includes windows) = 142.24m² (2dp)
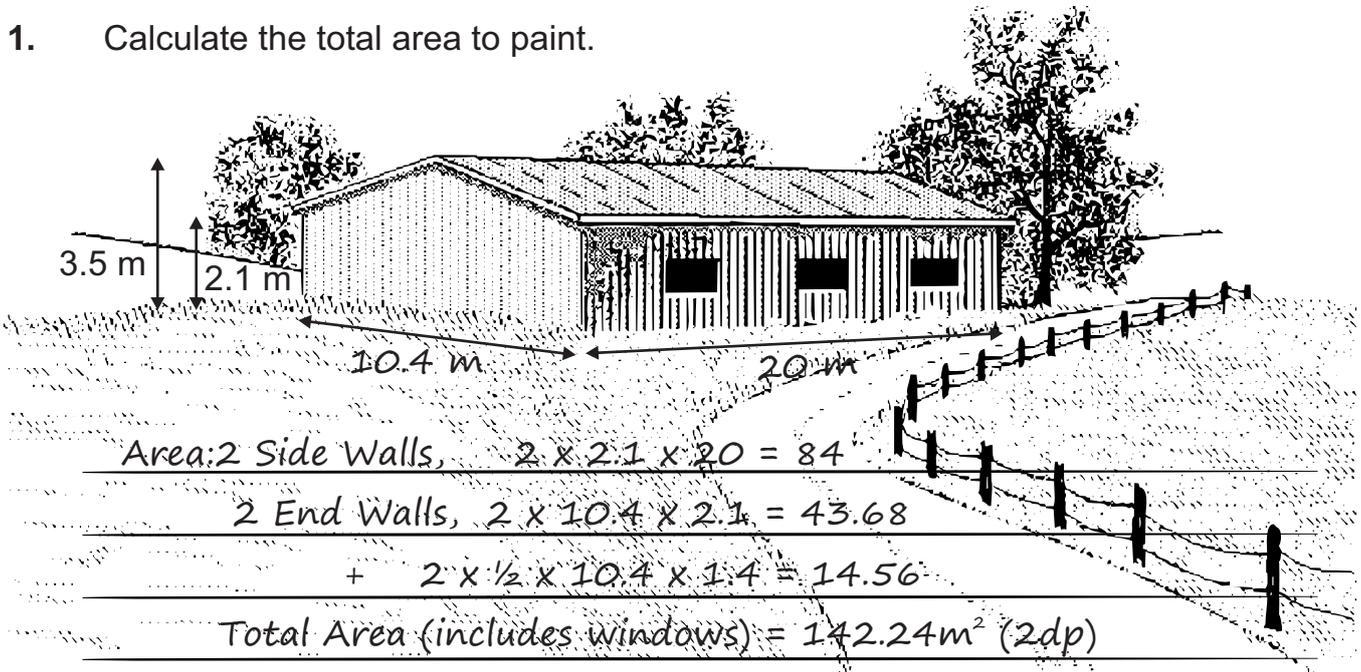
Subtract area of 6 windows (6 × 2 × 1 = 12 m²)

Total are to paint: 142.24 – 12 = 130.24m²

2.   A litre of paint covers 13 m². How many litres of paint are required for the job? (Assume that they only give the barn one coat of paint.)

130.24 ÷ 13 = 10.02 litres

3.   A litre of paint costs $39. The shop gives a 15% discount if 10 litres or more paint is purchased. How much will the paint cost?

Assuming 10 litres of paint was purchased = $390

$390 less 15% is 390 × 0.85 = $331.50

4.   You are paid $1000 to complete the painting job and you decide to enlist the help of 2 friends. You need to also purchase 3 paint brushes at $17.75 each. The job takes the three of you 2 whole days of working 8 hours per day to complete. Calculate your hourly rate (after paying for the paint and brushes).

Paint = $331.50, Brushes = $53.25, Total for material = $384.75

Profit: $1000 – $384.75 = $615.25

16 hours × 3 people = 48 man hours worked

$615.25 ÷ 48 = $12.82 per hour.

BLACK Worksheet 28

AN ECONOMICAL USE OF MATHS

The supply equation in economics is used to specify the amount of a particular commodity that sellers are willing to offer the market. The demand equation specifies the amount of a commodity that buyers are willing to purchase. An increase in price (p) usually causes an increase in the supply and a decrease in the demand. A decrease in price brings a decrease in supply and increase in demand.

Market price is defined as the price at which supply and demand are equal. Some supply and demand equations are straight lines while others are curved (similar to the illustration below).

Supply and Demand Equations
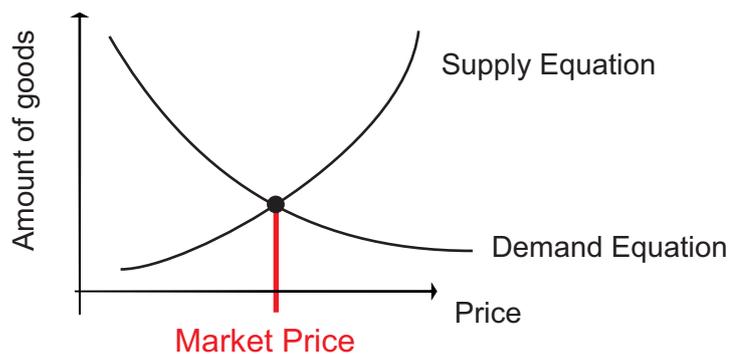
The supply and demand equations for flour has been estimated by economists to be given by the equations below. The variable p is measured in dollars ($) and S and D are measured in kg of flour.

$$\text{Supply} \qquad S = 0.8p + 0.5$$
$$\text{Demand} \qquad D = -0.3p + 1.5$$

1.   Find the market price of the supply and demand equations by graphing them on the grid below.

Supply and Demand Graph for Flour

Market price of flour
= $0.90 (90 cents)
for just over 1.2 kg
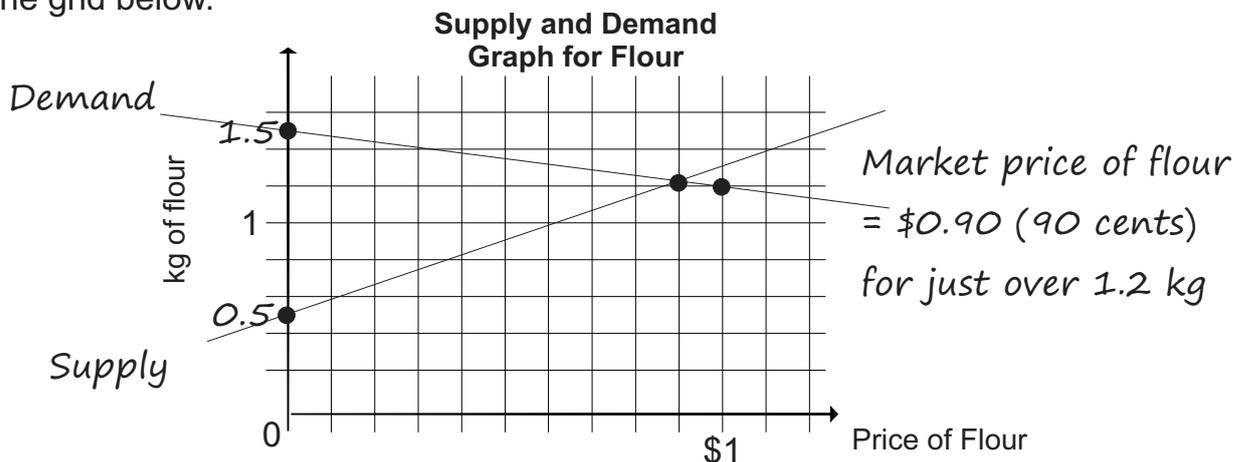
Note: Normally Economics graphs measure in 1000 units

AREA OF AN OCTAGON

Georgia wants to make a regular octagon.
Each side of the octagon will be 12 cm in length.

She finds a square piece of card and forms the octagon by cutting off the four corners.
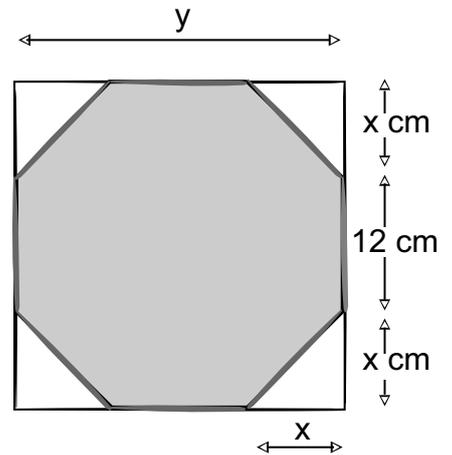
1. Write down an expression for the shaded octagon shown in terms of x and y.

$$\text{Area of the 4 triangle corners} = 4 \times \frac{1}{2} x^2$$

$$= 2x^2$$

$$\text{Area of the square} = y^2$$

$$\text{Therefore Area of Octagon} = y^2 - 2x^2$$

2. Use Pythagoras's Theorem to find the value of x.

$$x^2 + x^2 = 12^2$$

$$2x^2 = 144$$

$$x^2 = 72$$

$$x = 8.485 \ (3 \ dp)$$

3. Calculate the area of the octagon.

1. Square side length = $(8.485 \times 2) + 12$

   Therefore $y = 28.97 \ (2 \ dp)$

2. Area of Octagon = Square Area − Area of Corners

   $$= 28.97^2 - 2 \times (8.485)^2$$

   $$= 695.27 \ cm^2 \ (2 \ dp)$$

REAL LIFE APPLICATION - RIVER SPEED

Surveyors measured the speed of a current below the dam at Lake Karapiro. Based on their data the speed (S) in metres per second can be approximated by the formula:
$S = -0.03d + 0.46$, where d is the depth (in metres) of the water.

1. Describe what happens to the speed of the current as the depth of the water increases.

Try some data e.g. 1 m, 5 m, 10 m and the equation $S = -0.03d + 0.46$

1m, S = 0.43,   5m, S = 0.31,   10m, S = 0.16

As the depth increases, the speed of the water decreases.

LINEAR GRAPHS - HEIGHTS

The World Almanac has a section that deals with ideal height and weights for adults with a medium build. A person with height of 1.5 metres should have a weight of 51 kg. A person with height of 1.7 m should have a weight of 62 kg.

   i.   Graph the figures and write a linear equation to estimate the weight of a person of any height.

   ii.   Use your graph to estimate the ideal weight of a person who is 1.8 metres tall.

Graph of Ideal Heights
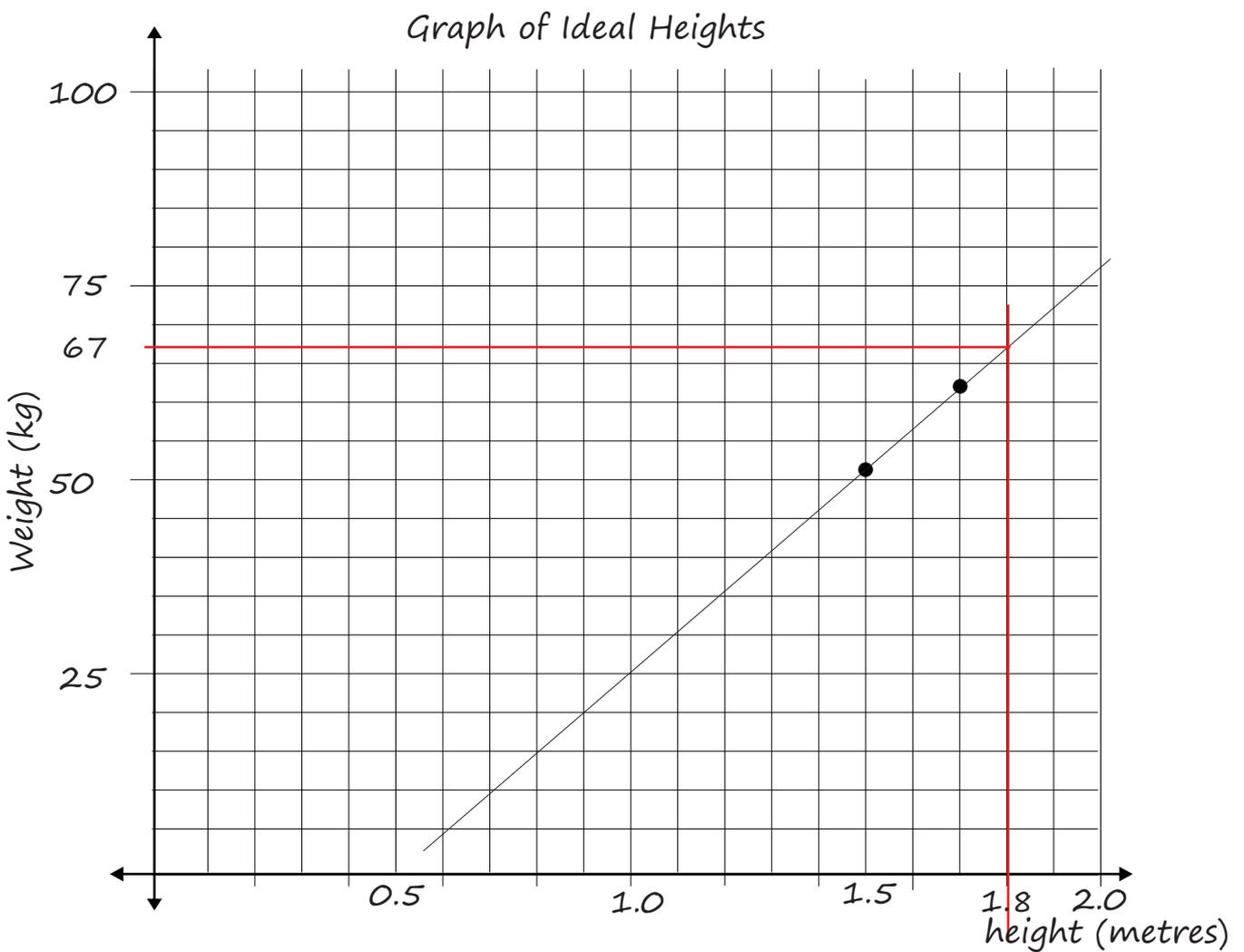
The accuracy of the estimate will depend on the scale of your graph. In the graph above, the best estimate is around 67 kg

MEASUREMENT APPLICATIONS

A yacht is to follow a triangular course from the start S, around a buoy B, then a rock outcrop R, and back to S. The buoy is placed 950 m from S so that angle SBR is a right angle. The rock outcrop is 2900 m from the start.

1. Calculate the total distance around the course, rounded to 2 sf.

Using Pythagoras, $2900^2 = BR^2 + 950^2$,

$BR = 2739.98$ m

i.e. BR = 2700 (2 sf), as all other figures are 2 sf

Total distance = 6550 m (2 sf)
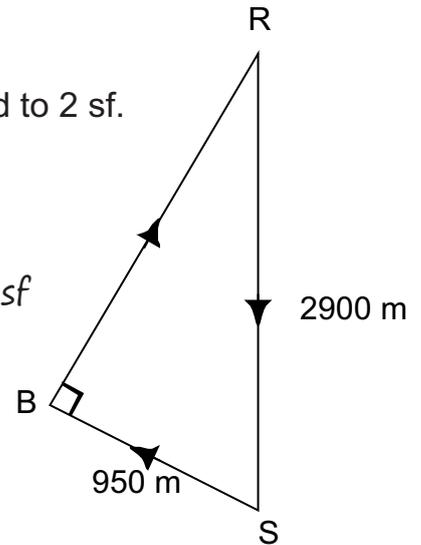
A kitchen shelf is made from a 50 cm square piece of wood. The horizontal base of the shelf is a semicircle and it is fixed to the wall by a support cut from the same piece of wood and shaped as a right angled triangle. The two diagrams below show the original drawings on the wood to be cut and the final shelf.
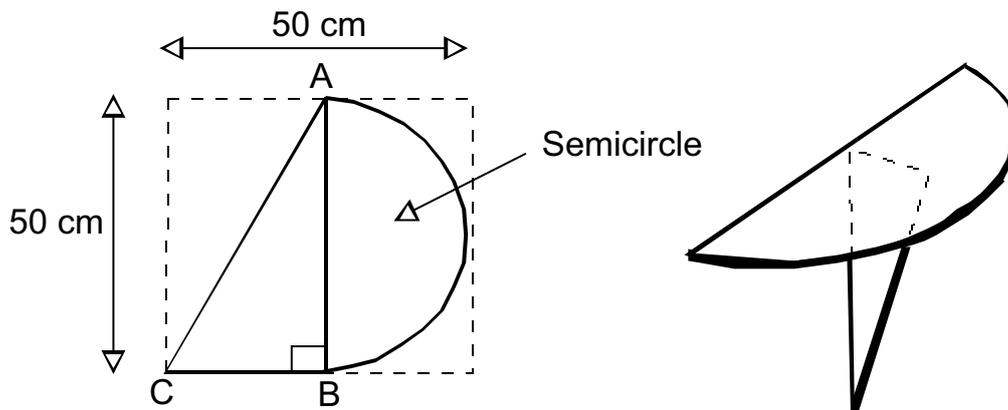
2. Use the plan to calculate the horizontal shelf area.

Area = $\frac{1}{2}\pi r^2$

$\frac{1}{2} \times 3.14 \times 25^2 = 981.25$ cm²

3. Use the plan to calculate the total area of wood used (i.e. the vertical support area + the horizontal shelf area).

Area triangle support is $\frac{1}{2} \times 50 \times 25 = 625$ cm²

Total Area is $981.25 + 625 = 1606.25$ cm²

4. Calculate the hypotenuse length of the vertical support (AC).

$AC^2 = CB^2 + AB^2$ $AC^2 = 25^2 + 50^2$

$AC^2 = 3125$, therefore AC = 55.90 cm

BLACK Worksheet 29

MATHEMATICS AND THE VALUE OF MONEY

To find the interest amount paid on borrowed money you use the formula I = PRT where I is the interest, P is the principal amount borrowed ($), R is the interest rate (%) and T is the time (years) of the loan.

1. Lauren borrows $5000 for 12 months at 6% per annum. How much interest does she pay?

    $I = 5000 \times 0.06 \times 1$

    $Interest = \$300$

2. James borrows $8000 for 5 months at 6.5% per annum. How much interest does he pay?

    $I = 8000 \times 0.065 \times \dfrac{5}{12}$

    Note how the loan is for $\dfrac{5}{12}$ of the year

    $Interest = \$216.67$

If a principal amount (P$) is borrowed at an interest rate (R%) for T (years) the future value of the loan (FV) can be calculated by the formula below.

FV = P(1 + RT)

3. Mika takes out a loan for $2000 to purchase a computer. The annual interest rate is 7% and she must repay the interest after 1 year. Find the future value of the loan after 1 year.

    $FV = 2000 \times (1 + 0.07 \times 1)$

    $FV = \$2140$

4. Mika decides to repay the above loan after 9 months. Find the future value of the loan that she will need to repay.

    $FV = 2000 \times (1 + 0.07 \times \dfrac{9}{12})$

    Note how the loan is for $\dfrac{9}{12}$ of the year

    $Interest = \$2105$

HOW TO MEASURE TALL STRUCTURES

The tallest man-made structure is the Burj Khalifa, in Dubai that is 828 m in height. The CN Tower in Toronto, Canada, (553.3 m) was formerly the world's tallest (1976 - 2007). The Petronius Platform stands 610 m off the sea floor leading some to claim it as the tallest freestanding structure in the world. Taipei 101 in Taipei, Taiwan, is 509.2 m and the Shanghai World Financial Center is 487 m in height. By comparison, the Sky Tower in, Auckland is 328 metres high. There is discussion over how to measure the tallest structures as some figures include tall radio masts on the top of the buildings.

| Burj Khalifa | CN Tower | Petronius Platform | Taipei 101 |

In this exercise you are going to measure the height of a tall building or structure. It may be a two or three storey building at school, a building in your town or a tall tree.

BUILDING AND USING A CLINOMETER

1. On the next page is an outline for a clinometer.
   You need to print 2 copies of this and glue it to two sides of card.

2. Tape a straw to the top of the card.

3. Thread some string through the middle and hang a weight from the string.

4. Work with a partner. Stand 10 metres from the base of the structure that you are measuring and sight the top of the structure through the straw. Do this 3 times with your partner measuring the angle on the clinometer each time.

5. Take another 3 set of measurements at 20 metres and 30 metres from the base of the structure.

6. Now use trigonometry to calculate the height of your structure.

straw   tape

The New Zealand Centre of Mathematics
www.mathscentre.co.nz

weight

CLINOMETER OUTLINE

90
80
70
60
50
40
30
20
10
0
10
20
30
40
50
60
70
80
90

The New Zealand Centre of Mathematics
www.mathscentre.co.nz

tape

straw

90
80
70
60
50
40
30
20
10
0
10
20
30
40
50
60
70
80
90

The New Zealand Centre of Mathematics
www.mathscentre.co.nz

weight
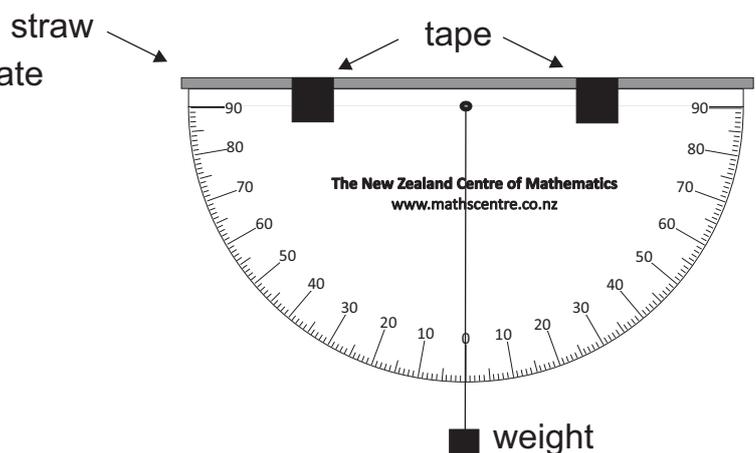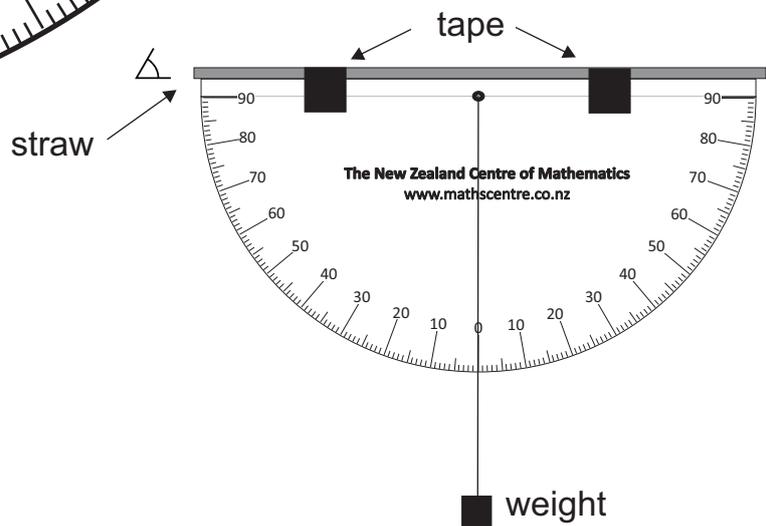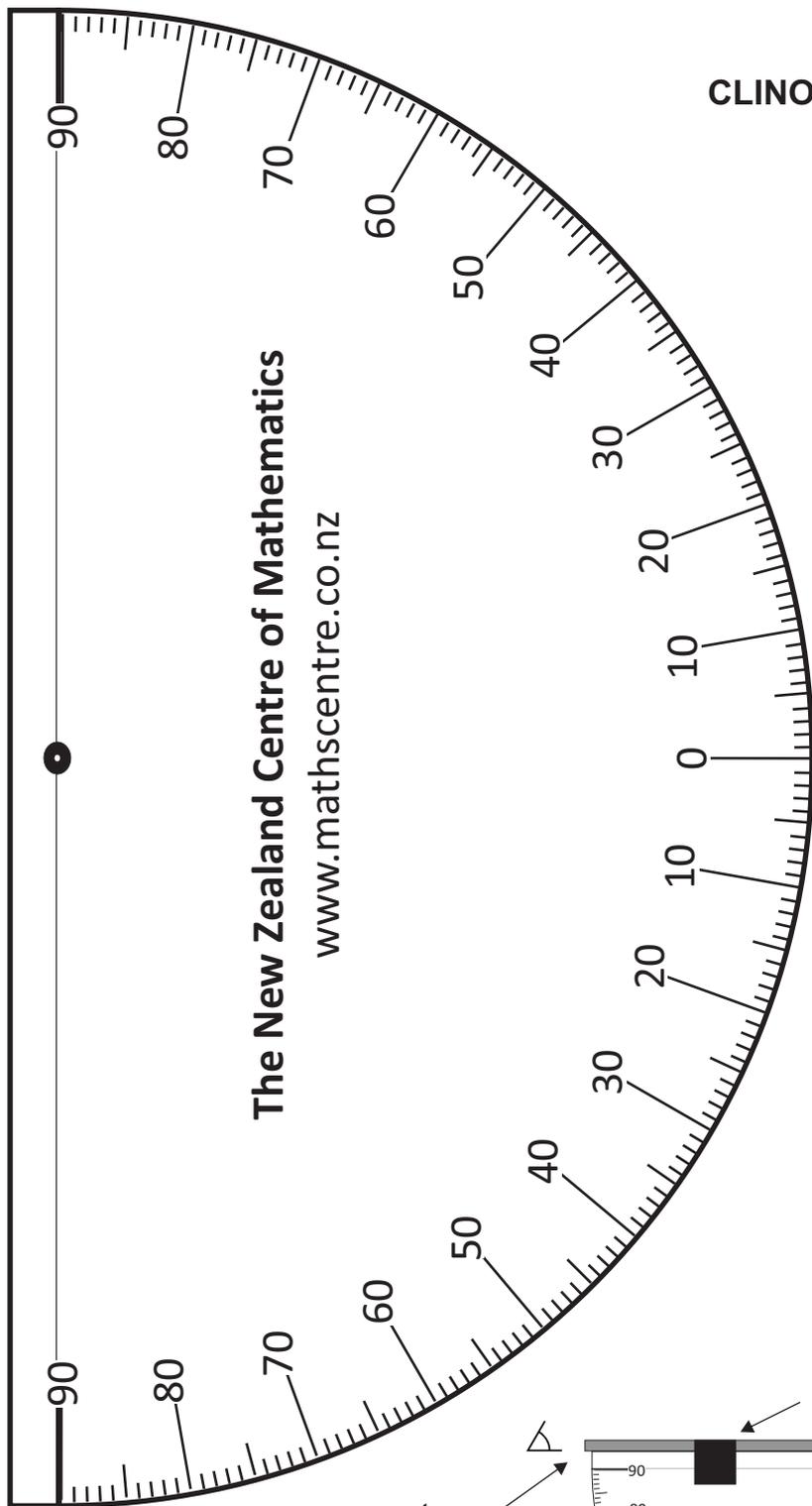
	1st measurement 10 metres	2nd Measurement 20 metres	3rd Measurement 30 metres
Angle 1			
Angle 2			
Angle 3			
Average			

Eye Level Height _____

Tan _____ × 10 = _____

Tan _____ × 20 = _____

Tan _____ × 30 = _____

Total = _____

Average = _____

+ Eye Level Height _____
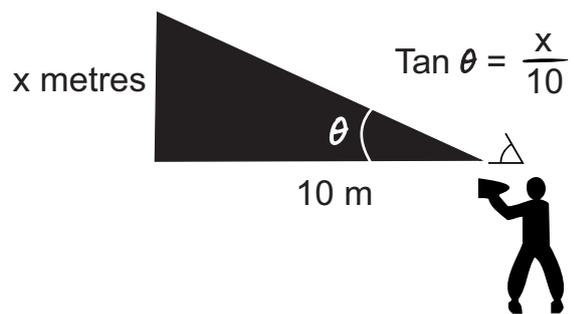
x metres

$\text{Tan } \theta = \dfrac{x}{10}$

θ

10 m

Stand 10 metres from the base of the structure that you are measuring and sight the top of the structure through the straw. Do this 3 times with your partner measuring the angle on the clinometer each time.

Take another 3 set of measurements at 20 metres and 30 metres from the base of the structure.

3 measurements at 3 different points ensures accuracy.
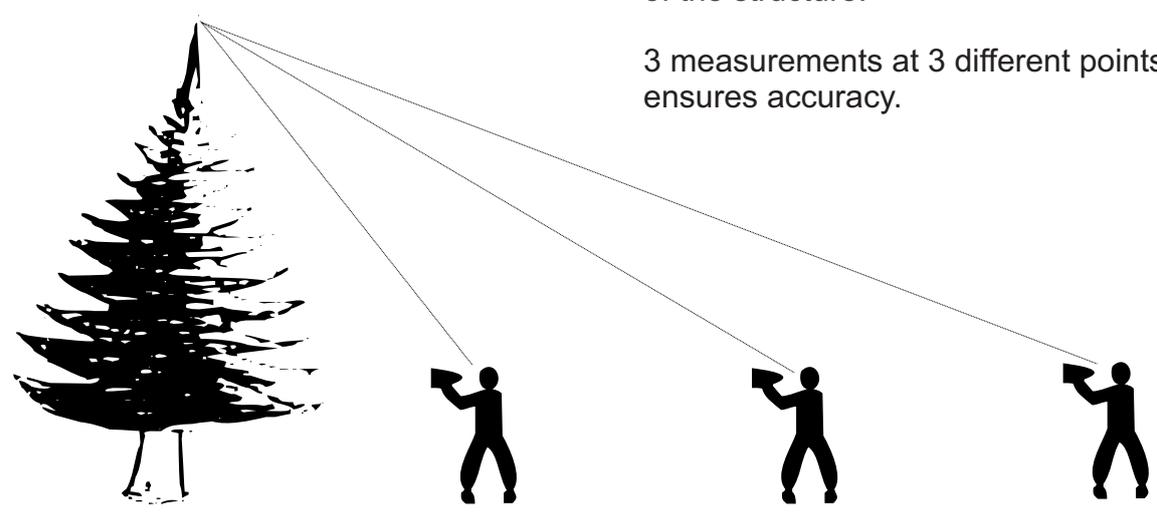
BLACK Worksheet 30

USING YOUR CALCULATOR

1. $6.5^2 - 3.5^2$ = _____30_____

2. $\dfrac{216 \times 0.015}{0.04}$ = _____81_____

3. $\dfrac{30}{3.5} + \dfrac{8}{1.6}$ = _____13.57_____

4. $\dfrac{3}{4}$ of 40 + $\dfrac{1}{4}$ of 60 = _____45_____

5. $(20 \times 15.5) - (16 \times 20.25)$ = _____−14_____

6. $\dfrac{35.72 + 61.8}{21.2}$ = _____4.6_____

7. $(5.5 \times 6.8)^2$ = _____1398.76_____

8. $\sqrt{4.84 \times 10^6}$ = _____2200_____

9. $(6.5 \times 10^{-3}) \div (7 \times 10^{-4})$ = _____9.29_____

10. $\sqrt[3]{\dfrac{360}{0.0748 \times 65.25^2}}$ = _____1.04_____

CALCULATOR CHALLENGES

1. $A = \sqrt{x(x-a)(x-b)(x-c)}$

a = 3.7
b = 4.4
c = 5.1

$x = \frac{1}{2}(a + b + c)$

Find A (accurate to 1 DP)

$x = \frac{1}{2}(3.7 + 4.4 + 5.1)$

$x = 6.6$

Using a calculator

$A = 7.9$

2. $V^3 = \dfrac{64P}{wA}$

P = 2450
w = 65.2
A = 0.0205

Find V (accurate to 1 DP)
Using a calculator

= 49.0

A DILEMMA THAT URGENTLY NEEDS TO BE CALCULATED

When Kate and Graeme were married they didn't have much money. On their first wedding anniversary Graeme was unable to give Kate a present. Instead he gave her 10 cents and promised to double the amount each year. Kate is looking forward to her silver wedding anniversary when they have been married for 25 years. However Graeme is not so enthusiastic. Why?

Use either a calculator or a spreadsheet. Graeme will have to pay $1,677,722

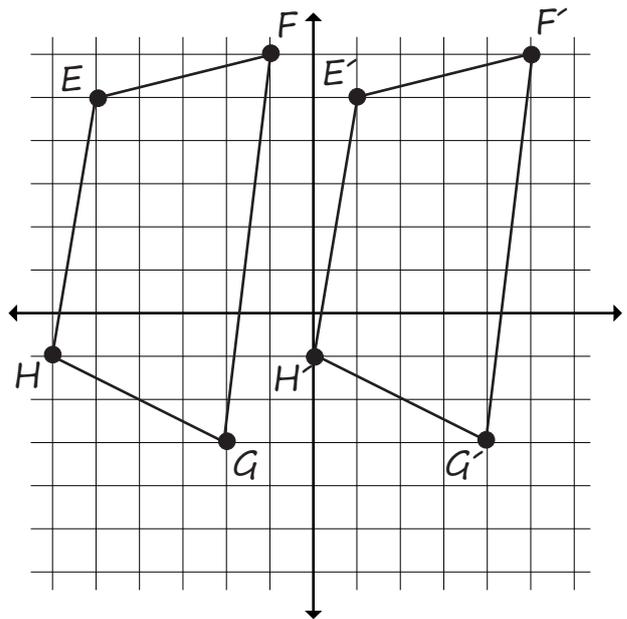# TRANSFORMATIONS

Draw the following figures and their images.

1. Draw triangle ABC.
A(4, -3), B(-2,-1), C(-2, -5)
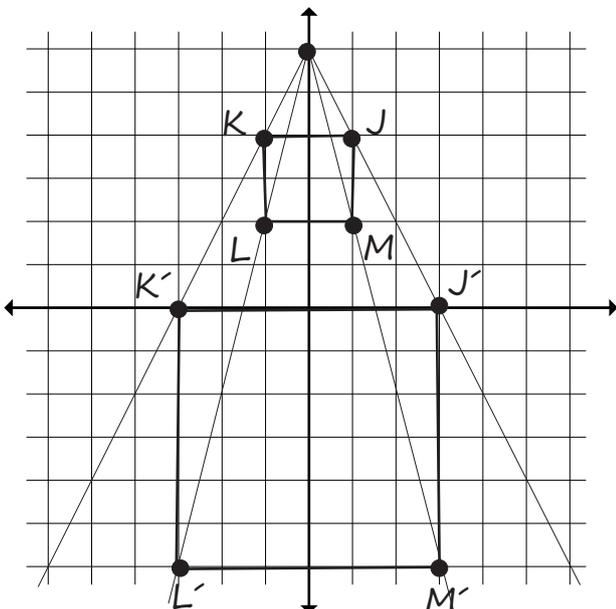
Reflect ABC in the x - axis.
Draw the image as A´B´C´
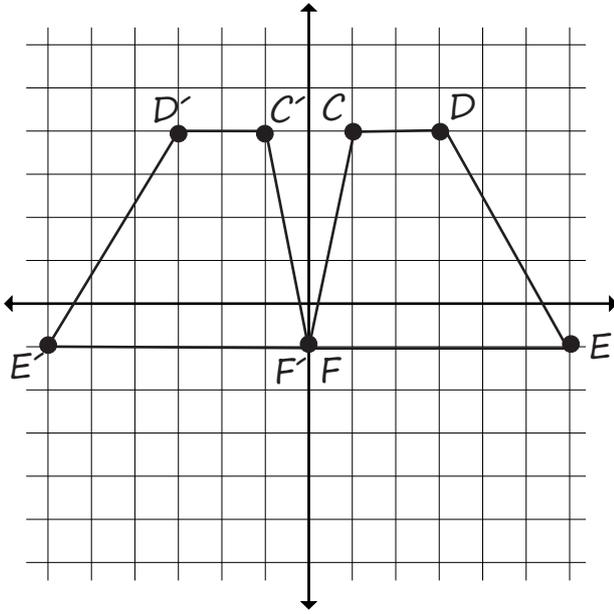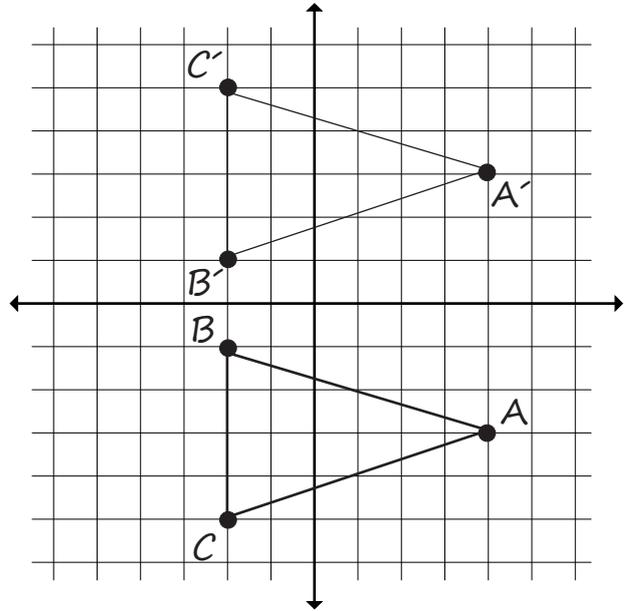
2. Draw trapezoid CDEF.
C(1, 4), D(3, 4), E(6, -1), F(0, -1)

Reflect CDEF in the y axis
Draw the image as C´D´E´F´.

3. Draw quadrilateral EFGH.
E(-5,5), F(-1, 6), G(-2, -3), H(-6, -1)

Translate EFGH 6 units right.
Draw the image as E´F´G´H´.

4. Draw square JKLM
J(1,4), K(-1, 4), L(-1, 2), M(1, 2)

Enlarge JKLM by scale factor 3
Use centre of enlargement (0,6)
Draw the image as J´K´L´M´.

LINEAR GRAPHS - METEOROLOGY

In a thunderstorm you normally see lightening, then after a short time you hear the thunder. The distance d in kilometres that sound travels can be expressed by the equation d = 0.34t (t is in seconds).

1.   Graph the equation then use your graph to estimate how long it would take to hear the thunder from storms that are 5 km and 8 km away.

Graph of Thunder Data
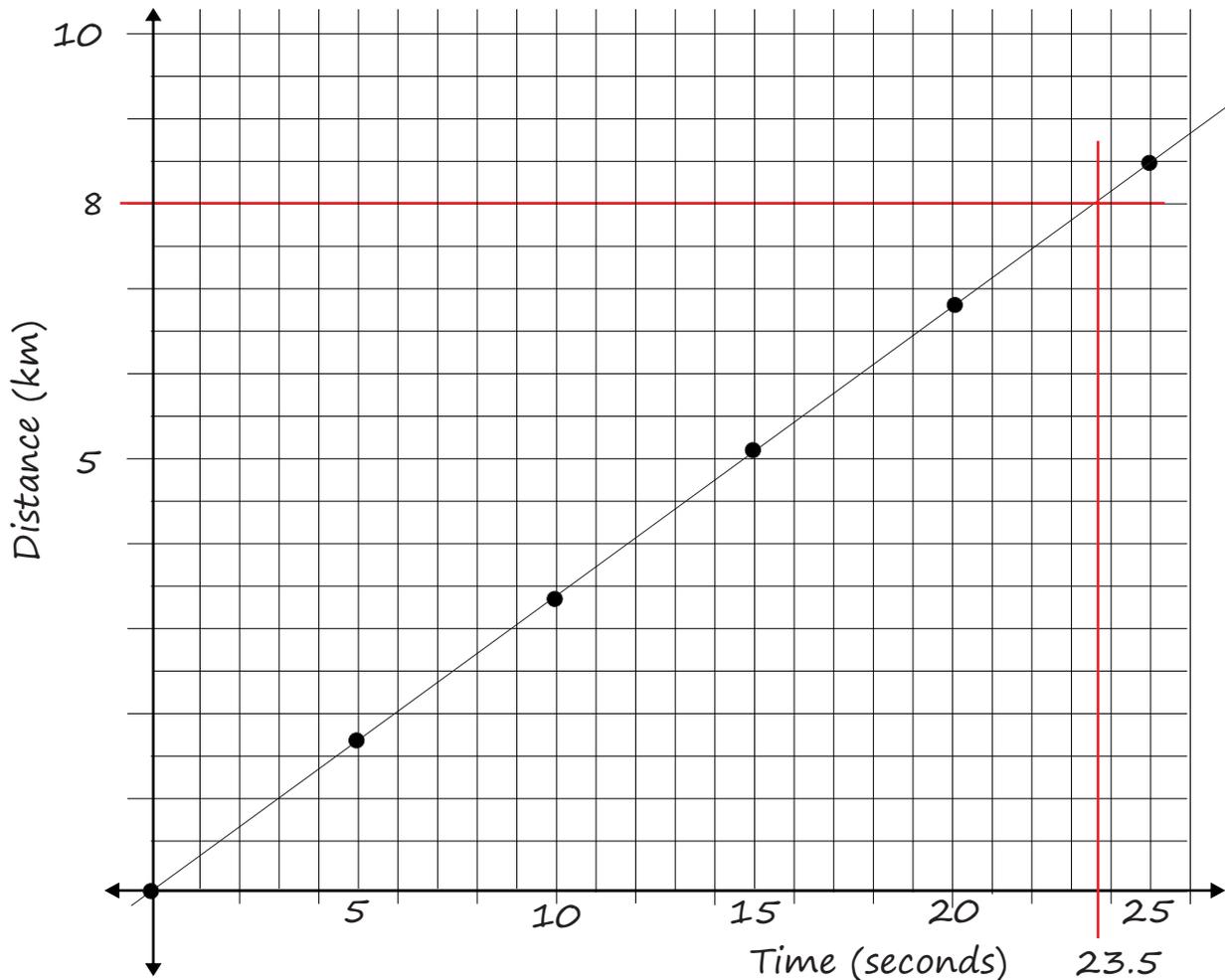
TIME (SEC)	0	5	10	15	20	25
DISTANCE (KM) D = 0.34t	0	1.7	3.4	5.1	6.8	8.5

The time taken before you hear a thunder clap 8 km away would be 23.5 seconds

GET THE POINT

The diagram below shows the graph of y = ½x - 3.

The graph shows the line y = ½x - 3 passing through points (0,-3) and (6,0), and the line y = -2x + 2.
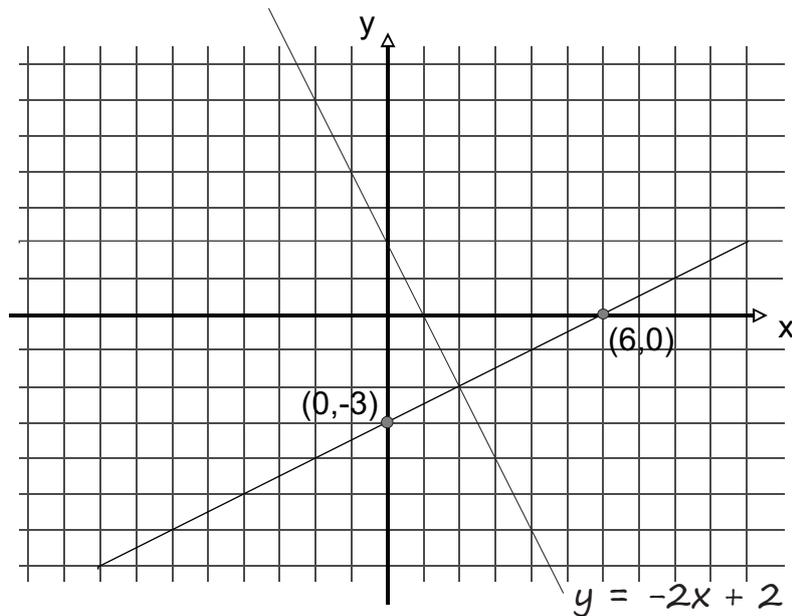
1. What is the gradient of this line? _____ *gradient = ½* _____

2. On the diagram, draw the graph of y = -2x + 2.

3. Using the graph, estimate the point of intersection of the lines:

y = ½x - 3 and y = -2x + 2 _____ *point = (2, -2)* _____

4. Simplify **i.** $\dfrac{3x^6}{x^2}$ = *3x⁴*

ii. 5x + 2 − (5 − 6x) = *11x − 3*

5. Expand and simplify (x + 2)(x − 7) = *x² − 7x + 2x − 14*

= *x² − 5x − 14*

6. Completely factorise 3x² − 9x = *3x(x − 3)*

7. Solve the equations: i. 6x + 5 = 50 *6x = 45*

x = 7.5

ii. x² + 14x + 48 = 0 *(x + 8) (x + 6) = 0*

i.e. x = −8 or x = −6

BLACK Worksheet 31
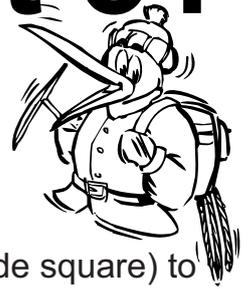
PROBLEM SOLVING

1.   What is the ratio of the area of the circumscribed square (the outside square) to that of the inscribed square (the shaded square inside)?
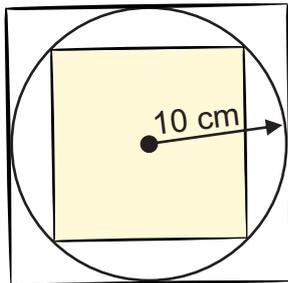
The diagonal of the small square is 20 cm

If each side is x cm then using Pythagorus

$x^2 + x^2 = 20^2$

Therefore $2x^2 = 400$

and $x^2 = 200$ cm² (the area)

The length of the larger square is 20 cm

Therefore the area is 400 cm²

This means that the ratio is 2 : 1

10 cm

2.   The volume of a cylinder is shown below. If the height of the cylinder is halved and the diameter is doubled, how will the volume be changed?

Try some simple figures

e.g. 1. d = 10 cm, h = 20 cm

*   2. d = 20 cm, h = 10 cm*

Volume 1 = 3.14 × 5² × 20

*   = 1570 cm³*

Volume 2 = 3.14 × 10² × 10

*   = 3140*
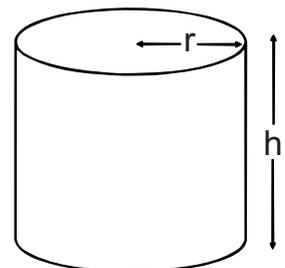
The volume will be doubled.

$$V = \pi r^2 h$$

3.
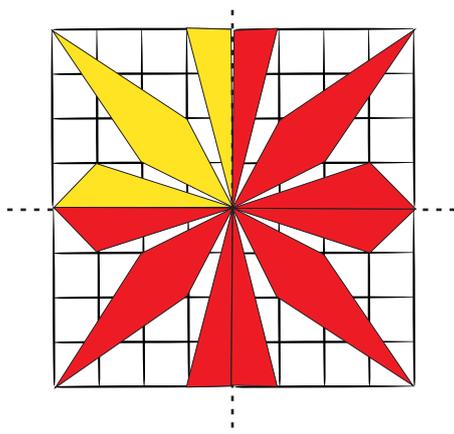
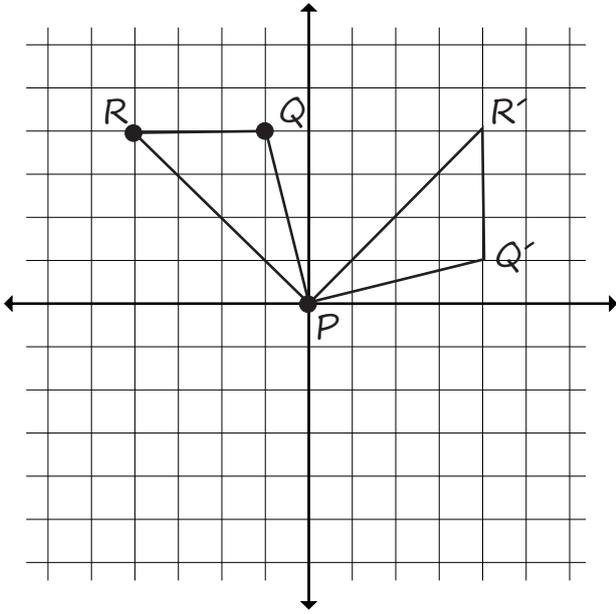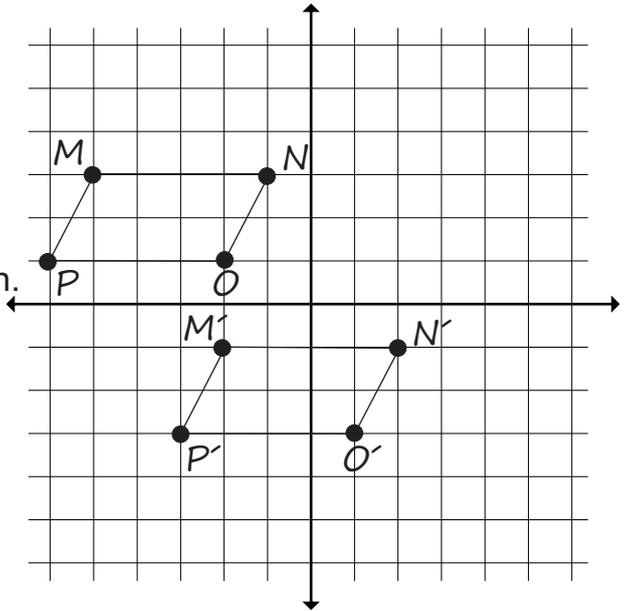The shape pictured is one quarter of a symmetrical pattern. The dotted lines are the axes of symmetry.

Complete the shape.

TRANSFORMATIONS

Draw the following figures and their images.

1. Draw parallelogram MNOP.
M(-5, 3), N(-1, 3), O(-2, 1), P(-6, 1)

Translate MNOP 3 units right, 4 units down.
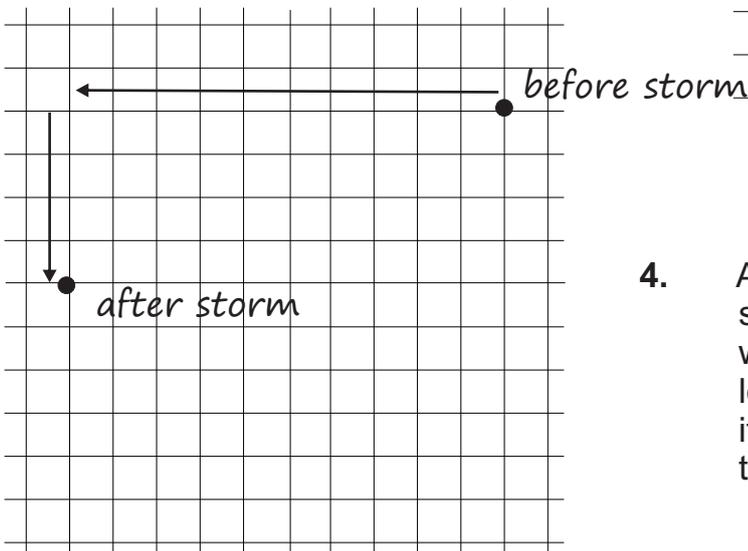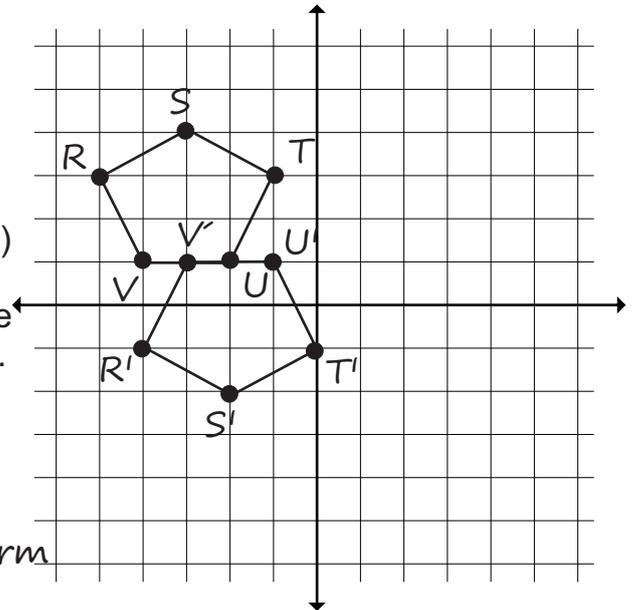Draw the image as M´N´O´P´.

2. Draw triangle PQR.
P(0, 0), Q(-1, 4), R(-4, 4).

Rotate PQR 90° clockwise around
the point (0, 0)
Draw the image as P´Q´R´.

3. Draw pentagon RSTUV
R(-5, 3), S(-3,4), T(-1, 3), U(-2, 1), V(-4, 1)

Reflect RSTUV in the x axis then translate
the pentagon 2 units up and 1 to the right.

4. A ship is blown off course by a
storm. After the storm it is 10 km
west and 4 km south of its original
location. Use the grid to show where
it is in relation to where it was before
the storm.

Assume each square unit represents 1km²
BLACK - Worksheet 31, Page 2

ANALYSING STATISTICAL DATA

John is surveying the smoking habits of the students in his year level. He can use one of two methods to gather the information he needs.

Method 1: *Ask students the questions face to face.*
Method 2: *Give students a form to fill out. The form does not ask for names.*

1.   Which method is the best to use? Give a reason.

Method 2 - the form to fill out. Students who were not required to give their name would be more likely to give honest answers.

One of the questions that John asks in his survey is "How many cigarettes did you smoke last Wednesday?" Here is his table of results:

| Number of cigarettes	0	1	2	3	4	5	6	7	8	9	10
Number of students	25	3	4	5	4	2	0	0	0	2	0

2.   How many students did John survey?   *Number of Students: 45*

3.   What was the mean and median number of cigarettes smoked last Wednesday?

Mean =1.56, Median = 0

4.   If a student is chosen at random, what is the probability that he/she will smoke at least 4 cigarettes.

Probability = 0.18 (8 ÷ 45)

5.   If you were to draw a pie chart which shows the percentage of smokers and non smokers then what would be the angle for each?

Angle smokers $\frac{20}{45}$ × 360° = 160°, Non Smokers $\frac{25}{45}$ × 360° = 200°

John finds some interesting data on young people and smoking habits. The graph of his data is below.

6.   Using the graph, write your conclusions about smoking trends.

Between 1985-95, smoking became more popular with females. Since then it has declined but at a slower rate than with males. Male smoking increased between 1985-90 but has since slowly decreased. The most dramatic decrease was 1990-95.

% of school students who smoke

BLACK - Worksheet 31, Page 3
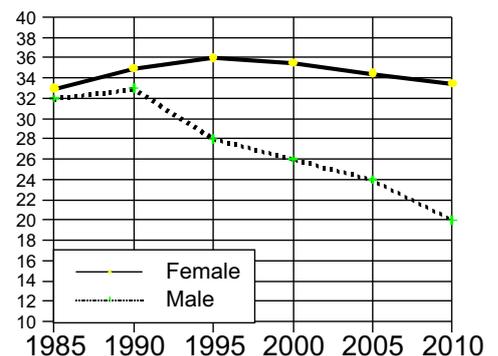
Knowing Number . . . . . complete these sentences.

1. BEDMAS means _calculations are done in this order: Brackets, Exponents, Division and Multiplication (in the order they occur) and finally Addition and Subtraction (in the order they occur)._

2. On a number line, positive numbers are on the __right__ of the zero, while negative numbers are on the left of the zero. The bigger the digit of the negative number, the __smaller__ the number.

3. 6^5 means 6×6 _$\times 6 \times 6 \times 6 = 7776$_

   $\left(\dfrac{2}{3}\right)^3$ means _$\left(\dfrac{2}{3}\right) \times \left(\dfrac{2}{3}\right) \times \left(\dfrac{2}{3}\right) = \dfrac{8}{27}$_

4. Equivalent Fractions are fractions which _have the same value_

5. To add or subtract fractions, the denominators must be _be the same_

6. To multiply fractions, multiply both numerators and denominators and then

   simplify the resulting fraction

7. To divide fractions, reverse the divisor fraction and _multiply_

8. To write a fraction as a decimal, divide the numerator by the _denominator_

   To write a fraction as a percentage, _multiply by 100_

   To write a percentage as a fraction, _divide by 100_

9. Numbers that are very small (e.g. 0.0065) or very big (e.g. 1 957 000 000) are written in _standard form_ (i.e. 6.5×10^{-3} and 1.957×10^9)

10. When rounding numbers, if the following digit in the series is ≥ 5 then you add __1__ to the last significant digit.

ASTRONOMY APPLICATION

1. The Sun burns approximately 4.4×10^6 tons of hydrogen per second. Using this information, how much hydrogen would the Sun burn in one year?
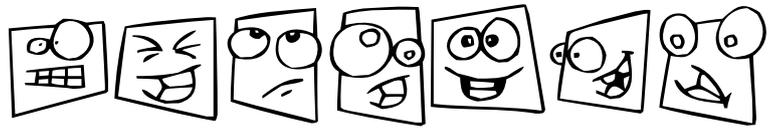
   Firstly find out how many seconds in 1 year.

   = 365 days × 24 hours × 60 minutes × 60 seconds = 3.1536×10^7

   $4.4 \times 10^6 \times 3.1536 \times 10^7 = 1.39 \times 10^{14}$ tones (140 trillion tons)

BLACK Worksheet 32

SEQUENCES OF NUMBERS

Complete the sequences:

1. $2,^{+3=}\ 5,^{+5=}\ 10,^{+7=}17,^{+9=}26,$ __37__ , __50__ , __65__ , __82__

2. $1,^{+4=}\ 5,^{+6=}\ 11,^{+8=}19,^{+10=}29,$ __41__ , __55__ , __71__ , __89__

3. $2,^{+5=}\ 7,^{+6=}\ 13,^{+7=}20,^{+8=}28,$ __37__ , __47__ , __58__ , __70__

4. $1,^{\times3=}\ 3,^{\times3=}\ 9,\ ^{\times3=}27,^{\times3}\ 81,$ __243__ , __729__ , __2187__ , __6561__

5. $1,^{\times2=}\ 2,^{\times3=}\ 6,^{\times4=}\ 24,^{\times5=}120,$ __720__ , __5040__ , __40 320__ , __362 880__

Find the values of:

6. $\sqrt{25 \times 144}$ $\sqrt{25}\ \times\ \sqrt{144}$ $\sqrt{25 + 144}$ $\sqrt{25}\ +\ \sqrt{144}$

   $=\sqrt{3600}$ $= 5 \times 12$ $=\sqrt{169}$ $= 5 + 12$

   $= 60$ $= 60$ $= 13$ $= 17$

7. Extend these number patterns:

$5^2,$	0	×	10	+	25	=	25
$15^2,$	10	×	20	+	25	=	225
$25^2,$	20	×	30	+	25	=	625

   $35^2, 30 \times 40 + 25 = 1225$

   $45^2, 40 \times 50 + 25 = 2025$

   $55^2, 50 \times 60 + 25 = 3025$

   $65^2, 60 \times 70 + 25 = 4225$

   $75^2, 70 \times 80 + 25 = 5625$

   $85^2, 80 \times 90 + 25 = 7225$

3	×	37	=	111
6	×	37	=	222
9	×	37	=	333

   $12 \times 37 = 444$

   $15 \times 37 = 555$

   $18 \times 37 = 666$

   $21 \times 37 = 777$

   $24 \times 37 = 888$

   $27 \times 37 = 999$

8. What is the value of x in the equation: $2^3 \times 3^x \times 5 = 1080$?

   $40 \times 3^x = 1080\ therefore\ 3^x = 27\ and\ x = 3$

Knowing Measurement, Shape & Space . . . . . complete these sentences.

Measurement: The most common unit for length is the _metre_

For smaller lengths use _centimetres_

For accurate small measures, use _millimetres_

10 mm = _1_ cm   100 cm = _1_ m   1000 m = _1_ km.

The standard unit for mass is the _kilogram_ For small measures, use _grams_ and for larger measures use _tonnes_

Capacity of an object describes how much air or liquid an object can hold. The basic unit for capacity is the _litre_ smaller measures (e.g. medicine) are measured in _millilitres (ml)_. Volume of an object is found by calculating the area of the object's main face and then multiplying by its _thickness (depth)_

Remember these measures: 1000cm^3 = _1_ litre, a bucket contains approximately _10_ litres, and a milk carton contains _1 litre_

The basic unit for temperature is _degrees celsius_ Water freezes at _(zero)0_ °C and boils at _100_ °C .

In the 24 hour clock system, the numbers are numbered from 0 to _24_ The first two numbers give the hours after _midnight_ the second two numbers give the _minute times_

Give the formulae for area and perimeter of a quadrilateral, a triangle and a circle.

Perimeter add all side lengths. A quadrilateral is a four sided polygon.
Area of a rectangle = L × B, Area trapezium ½(A+B) × H
Area triangle = ½BH
Area Circle = ¶2², circumference = ¶D

Pythagoras' Theorem states that _a² + b² = c² where a and b are sides of_
a right angled triangle, c is the hypotenuse

To help remember trig ratios, we learn the word _SOHCAHTOA_

Sine = $\frac{opposite\ side}{hypotenuse}$ Cosine = $\frac{adjacent\ side}{hypotenuse}$ Tangent = $\frac{opposite\ side}{adjacent\ side}$

THE MATHEMATICS OF SHAPE AND SPACE

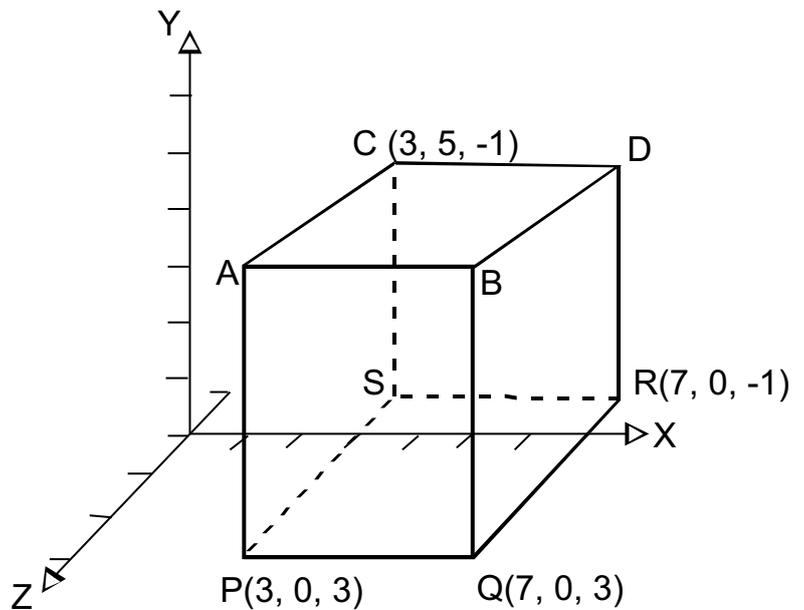The diagram shows a cuboid drawn on a set of 3D axes.

The axes have an x, y and a z axis. Co-ordinates are given as (x, y, z).

The coordinates of P are (3, 0, 3)
The coordinates of Q are (7, 0, 3)
The coordinates of R are (7, 0, -1)
The coordinates of C are (3, 5, -1)

1. Write down the (x, y, z) coordinates.

of A (__3, 3, 3__) and of D (__7, 5, -1__)

2. Write down the lengths of the following:

PQ __PQ = 4 units__

QR __QR = 4 units__

PR __PR = 5.66 units (2 dp) – using Pythagoras and PQ and QR__
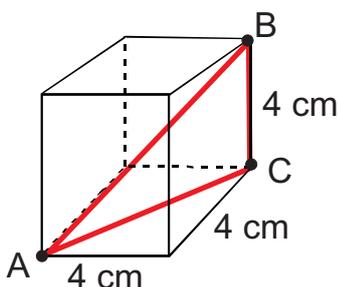
3. Calculate the total surface area of the cuboid.

Area of the Sides: Top & Bottom 4 × 4 × 2 = 32

Left and Right 4 × 5 × 2 = 40

Front and Back 4 × 5 × 2 = 40

Total Surface Area= 112 units²

4. The figure below shows a cube with sides of 4 cm. A right angled triangle (ABC) is drawn fitted within the cube. Find the length of AB.

AC² = 4² + 4²

AC² = 32 therefore AC = 5.66 cm

AB² = AC² + BC²

_AB² = 5.66² + 4²_

AB² = 48.04, therefore AB = 6.9 cm

ELASTIC STRETCH AND LINEAR GRAPHS

Class 10FR hang different weights from an elastic band. The length that the band stretches is then measured. After weighing a range of objects, they discover that the weight of the object (x gm) is related to the stretch of the band (y cm) by the equation $y = 3 + 2x$.

1. On the graph below, show how x and y are related.

$y = 2 + 4x$ $y = 3 + 2x$

length (centimetres)
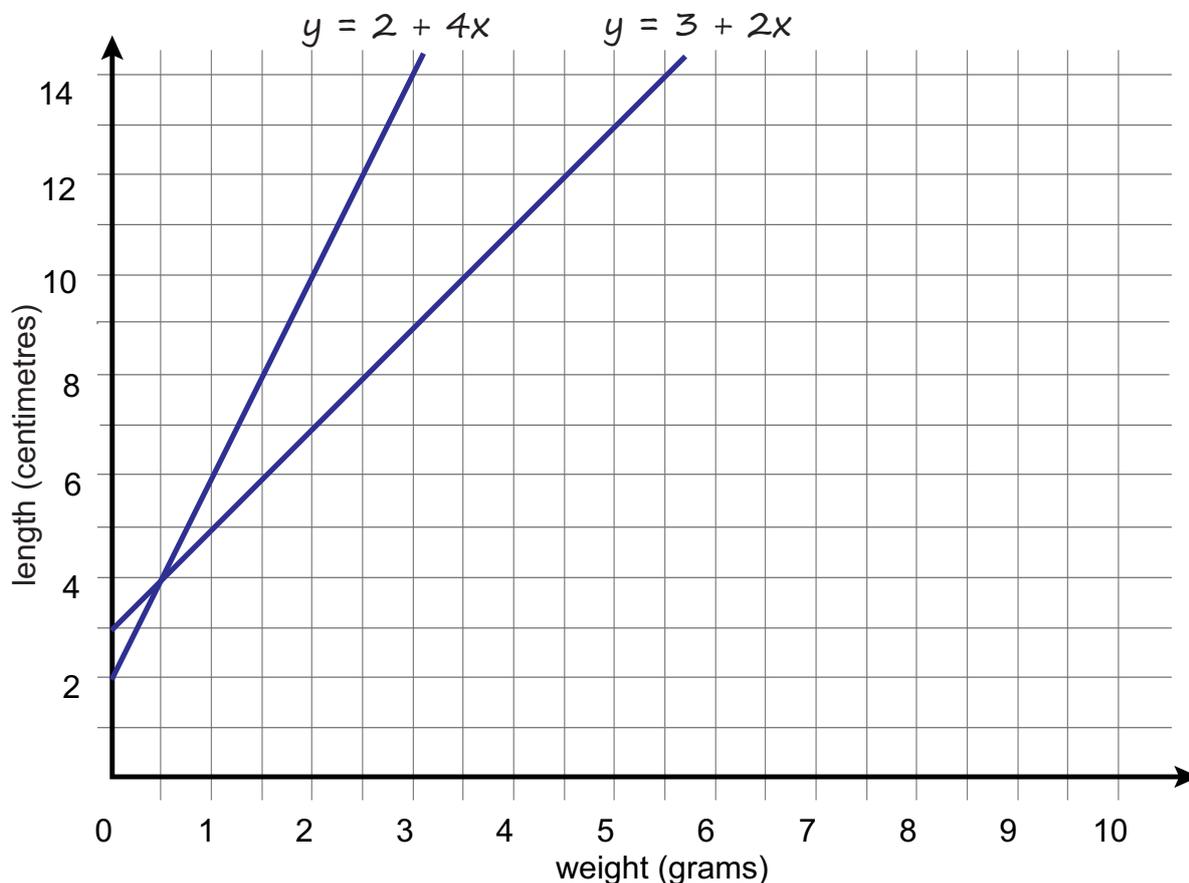
weight (grams)

2. Find the length of the elastic band when:

i. there is no weight.   _0 grams = 3 cm_

ii. 5.5 grams are hung from it.   _5.5 grams = 14cm_

3. Make x the subject of the formula $y = 3 + 2x$.

$$x = \frac{y - 3}{2}$$

Another elastic band is tried with the same weights. With the second band, the weight is related to the stretch by the formula $y = 2 + 4x$.

4. Find x and y when the lengths of the two elastic bands are the same.

Read from your graph: i.e. x = 0.5 g, y = 4 cm

BLACK Worksheet 33

QUICK CALCULATIONS

1. $18 \div 24 =$ *0.75*

2. $3 \div 1\frac{1}{2} =$ *2*

3. $0.4 \div 0.2 =$ *2*

4. $72 \div (6 \times 2) =$ *6*

5. $8808 \div 12 =$ *734*

6. $5 \div \frac{1}{2} =$ *10*

7. $9.45 \div 100 =$ *0.0945*

8. $45 \div 0.9 =$ *50*

9. $100^2 \div 10 =$ *1000*

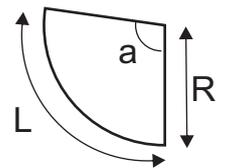10. $1000 \div 10 \div 5 =$ *20*

EQUATIONS TO SOLVE

1. $5x = 1.5$, $x =$ *0.3*

2. $2y + 8 = 14$, $y =$ *3*

3. $6z - 1 = 5z + 9$, $z =$ *10*

4. $4(j - 2) = 20$, $j =$ *7*

5. $k + 0.8 = 2.2$, $k =$ *1.4*

6. $m^2 = 6.25$, $m =$ *2.5*

7. $10p = 1$, $p =$ *0.1*

8. $7 + r \times 2 = 17$, $r =$ *5*

9. $9600 \div s = 240$, $s =$ *40*

10. $4u - 18 = 26$, $u =$ *11*

TRADE MATHEMATICS

The length of the arc of a sector of a circle can be found by using the formula:

$$L = \frac{2¶Ra}{360}$$

L = length of the arc
¶ = pi (3.14)
R = radius length
a = angle

1. A sheet metal worker has to design a cone which has a circular base. The radius (R) is 5 metres and the angle (a) is 30°. What will be the diameter of the base of the cone? (You also need to know the formula for the circumference of a circle.)

   $2¶Ra \div 360 = 942 \div 360$

   $= 2.62$ metres

   This is the circumference

   of the circle.

   Circumference $= ¶D$

   $2.62 = 3.14D$

   $D = 0.83$ metres

2. The cost of the material is based on the area of the metal used. Area of a sector is:

   $$A = \frac{¶R^2a}{360}$$

   Calculate the area of the sector above.

   $¶R^2a \div 360 = 2355 \div 360$

   $= 6.54$ metres^2

Knowing Probability and Statistics . . . . . complete these sentences.

Probability measures the likelihood of an event happening.

It is expressed as a fraction, decimal or percentage and will always be between ___0___ and ___1___ .

A probability of 0 means the event will ___not___ happen and a probability of 1 means the event is ___certain___ to occur.

Tree diagrams can help illustrate probabilities. Each branch of the tree shows the ___probability___ of the event and each set of branches must always add up to ___1___

Averages give a central value that is representative of a given set of numbers.

The three most common averages are ___mean___ , ___median___ , ___mode___

The mean is the sum of all the values ___divided___ by the amount of values in the group. The median is the ___middle___ number when they have all been placed in numerical order. The mode is the value that occurs the ___most___

As well as averages, numeric data can be analysed by calculating the measures of spread such as ___range___ and ___quartiles___ The range is found by taking the ___smallest value___ from the ___largest.___ Quartiles divide the group of numbers into ___4___ equal parts. To illustrate measures of spread, or to compare data from two or three sources, draw a ___box___ and ___whisker___ graph.

Time Series data is collected over a ___long___ period of time. It is then analysed in an attempt to find any ___trends___ so that we can make predictions.

MATHS WHILE EATING OUT

At a restaurant you can choose a first course, main course and dessert. You can choose soup or shrimp cocktail for the first course, steak, chicken, fish or vegetarian lasagne for the main course and chocolate mousse or fruit. Give the number of possible meals that can be chosen.

___There are 16 possible choices___

ICECREAM STATISTICS

The Creamy Ice Company sells various sorts of ice cream.
Here are the sales figures for one week in November:

	Mon	Tue	Wed	Thur	Fri	Sat	Sun
Iceberger	13	15	12	16	15	37	41
Triple Tub	12	17	15	15	12	42	45
Cee Ice	25	20	24	20	24	58	55
Creamy Choc	17	16	15	20	19	33	37

1. Which is the most popular ice cream? _Most popular ice-cream = Cee Ice_

2. Which is the most popular day for people to buy ice creams? _Sunday_

Last year, the Creamy Ice Company recorded the following sales:

Season	Autumn	Winter	Spring	Summer
Sales (thousands)	8.4	6.2	7.9	18.1

3. What were the total sales for the year? _Total sales = 40 600_

4. If this information was drawn on a pie chart, what would the angle of the Winter
   section be? $\frac{6.2}{40.6} \times 360° = 54.9°$

   Angle = 55° (2sf)

So far, the Creamy Ice Company has recorded the following sales this year:

Season	Autumn	Winter	Spring	Summer
Sales (thousands)	9.2	6.8	8.7	____

5. If you had to predict sales for summer this year, based on last year's sales
   figures, how many sales would you predict? Explain your answer.

   Summer= 19.9, Explanation: Sales for each season are

   approximately 10% better than the previous year.

The box and whisker graph shows an
analysis of the daily sales of each of
the different types of ice cream during
last October.

Daily Sales

6. Which ice cream had the
   largest range of sales?
   Explain how you come to this answer.

   Largest Range = Cee Ice

   It has the longest "box and whisker"
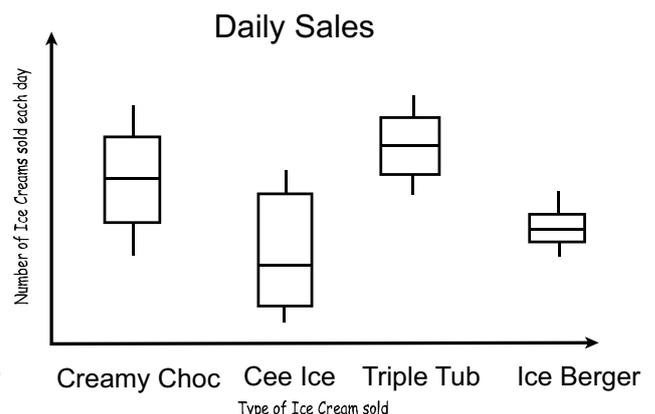
STATISTICAL APPLICATIONS

A consumer-watch magazine has given you the task of evaluating two kinds of battery. You test a batch of each battery in a portable DVD player and note how long (in hours) each runs.
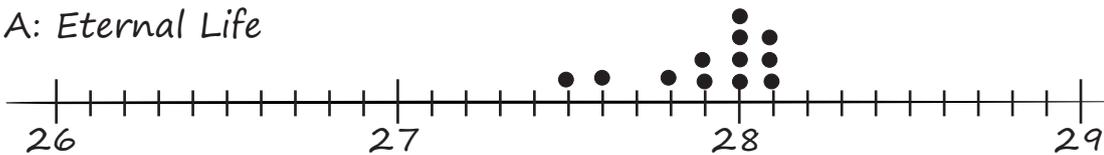
Battery Brand A - Eternal Life
27.5, 27.6, 27.8, 27.9, 27.9, 28.0, 28.0, 28.0, 28.0, 28.1, 28.1, 28.1
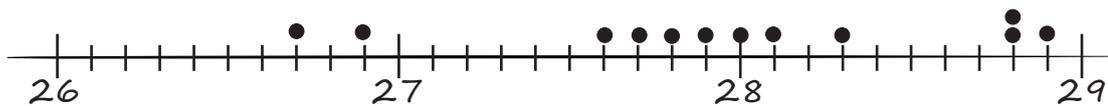
Battery Brand B - Super Cell
26.7, 26.9, 27.6, 27.7, 27.8, 27.9, 28.0, 28.1, 28.3, 28.8, 28.8, 28.9

1. Draw a dot plot, find the mean, median and write a statement that tells which you feel is the best battery.

A: Eternal Life

B: Super Cell

Mean (hours): Battery A = 27.92, B = 27.96

Median(hours): Battery A = 28, Battery B = 27.95

The data for Battery A is not as variable as Battery B. Both have (virtually) identical means and median. For a reliable battery go with Battery Brand A as the spread of data is 0.6 of an hour. Battery Brand B has a spread of 2.2 hours.

2. The distances (in centimetres) of 11 jumps from the final round of a women's long jump competition are listed here: 703, 702, 685, 677, 689, 670, 665, 669, 645, 687, 674. Display the data on a Stem and Leaf Plot to show how the data is distributed and circle the median and quartile values.

Woman's Long Jump Results

Unordered Plot		Ordered Plot	
64	5	64	5
65		65	
66	5 9	66	5 ⑨ LQ
67	7 0 4	67	0 4 ⑦ median
68	5 9 7	68	5 7 ⑨ UQ
69		69	
70	3 2	70	2 3

BLACK Worksheet 34

ROMAN NUMERALS

1. The Pantheon in Rome has been used in the past as a royal tomb and is still used today for church services. Inside the circular floor has an area of 1473 square metres. What is the diameter of the floor?

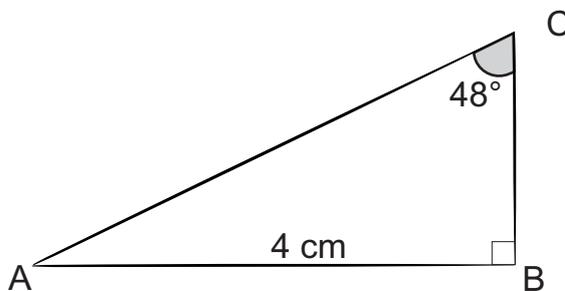$Area = πr²$ $(π = 3.14, r = radius)$

therefore $1473 = 3.14r²$

$r² = 469.11$

$r = 21.66$

therefore the diameter of the floor = 43.32 metres.

TRIGONOMETRY

Find each of the lengths AC.
Note that the triangles are not drawn to scale.

1.

$Sin\ 48° = \dfrac{4}{AC}$

$AC = \dfrac{4}{Sin\ 48}$

$AC = 5.38\ cm$

2.

$Cos\ 75° = \dfrac{AC}{12}$

$AC = Cos\ 75 × 12$

$AC = 3.11\ cm$

3.

$Tan\ 20° = \dfrac{AC}{10}$
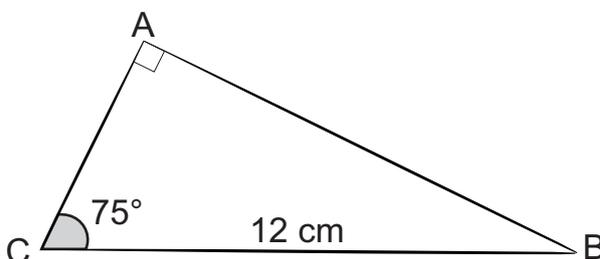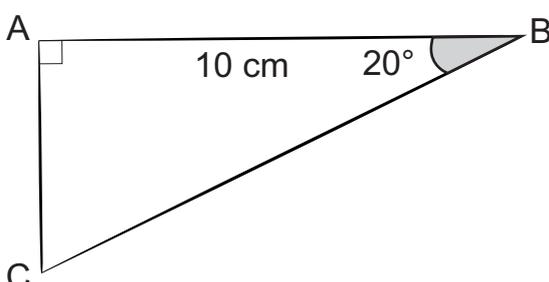
$AC = Tan\ 20 × 10$

$AC = 3.64\ cm$

Knowing Algebra . . . . complete these sentences.

When adding or subtracting algebraic equations only combine ___*like*___ terms.
When multiplying or dividing, multiply or divide the numbers then the ___*variables*___

When expanding algebraic equations, multiply any terms outside the brackets by those inside, then ___*simplify*___ by combining any like terms.

When factorising an expression, find a ___*common*___ factor, then rewrite the expression. Factorising quadratics usually involves some trial and ___*error*___

A formula is a general rule written as an ___*equation*___. To substitute values, replace the variables with the given ___*values*___. Rearranging a formula usually involves changing the subject of an equation. The subject of an equation is a single variable that comes before the ___*equals*___ sign.

When solving equations, you usually have to move variables or numbers from one side of the equation to the other. When doing this, change addition signs to ___*subtraction*___ and multiplication signs to ___*division*___.

EQUATION SOLVING

Solve each equation.
The domain is {-3, -1, 2, 5}.

1. $y = x - 10$

   $x = -3, y = -13$

   $x = -1, y = -11$

   $x = 2, y = -8$

   $x = 5, y = -5$

2. $y = 2x + 5$

   $x = -3, y = -1$

   $x = -1, y = 3$

   $x = 2, y = 11$

   $x = 5, y = 15$

3. $2x - y = -3$
   rewrite equation $y = 2x + 3$

   $x = -3, y = -3$

   $x = -1, y = 1$

   $x = 2, y = 7$

   $x = 5, y = 13$

4. $3x - \frac{1}{2}y = 6$
   rewrite equation $y = 6x - 12$

   $x = -3, -30$

   $x = -1, -18$

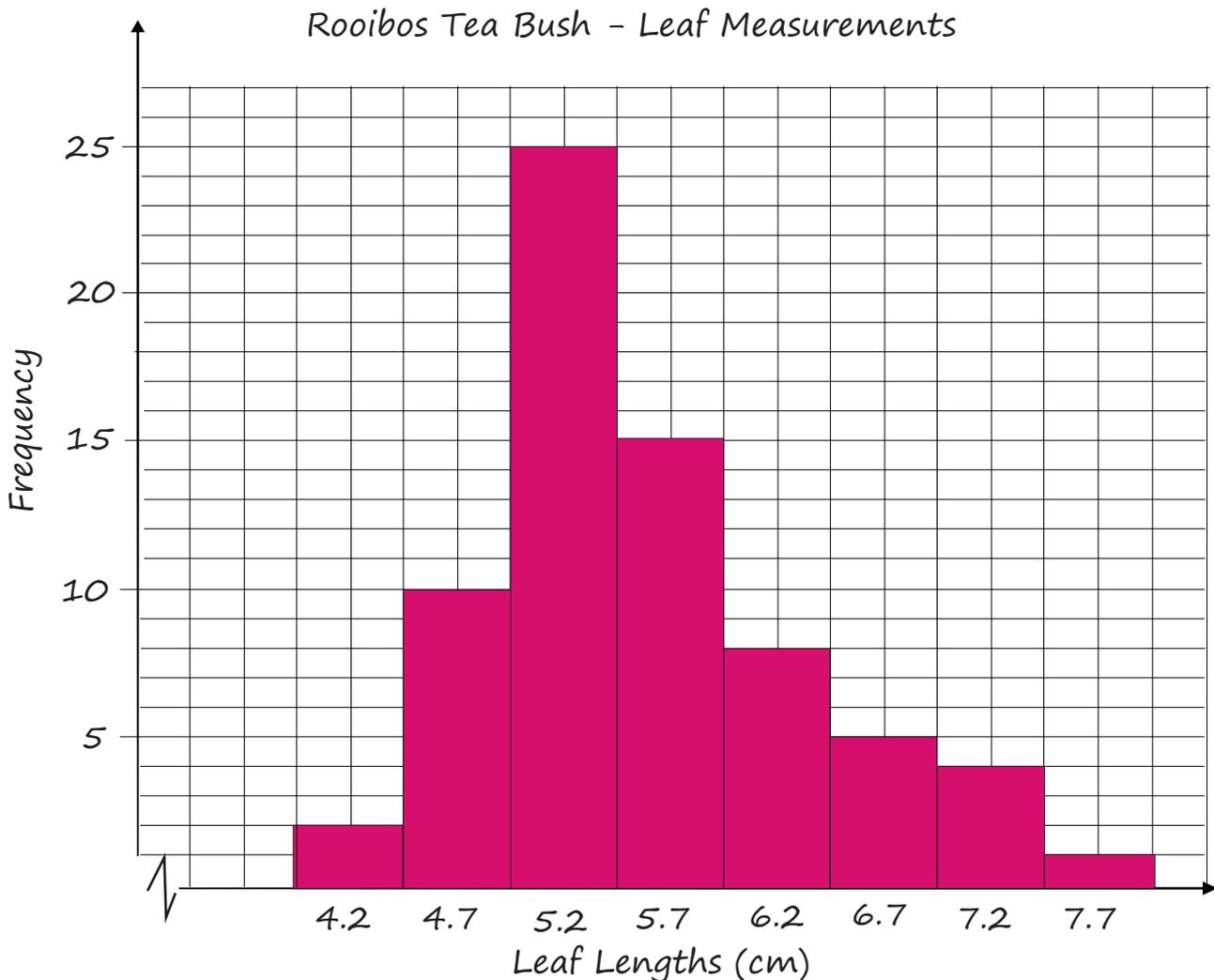 $x = 2, 0$

   $x = 5, 18$

STATISTICAL ANALYSIS

Below are the lengths of leaves taken and measured from
a Rooibos Tea Bush.

Length (cm)	4.0 - 4.4	4.5 - 4.9	5.0 - 5.4	5.4 - 5.9	6.0 - 6.4	6.5 - 6.9	7.0 - 7.4	7.4 - 7.9
Frequency (f)	2	10	25	15	8	5	4	1

Use the data collected to answer the following questions:

1. How many leaves were measured? _____ 70 _____

2. What is the mode of the data? _the 5.0-5.4 cm data measurements_

3. What is the median of the data? _median position = (x + 1) ÷ 2_
   _____ 71÷2 = 35.5
   the 5.0-5.4 cm data measurements

4. Draw a histogram of the data.
   Remember to give your histogram a title and label the axes.

Rooibos Tea Bush – Leaf Measurements

MORE STATISTICAL ANALYSIS

The lengths of leaves taken and measured from a Rooibos Tea Bush are now rewritten in a different form.

Length (cm)	Frequency (f)	Interval Mid-Point (x)	f × x
4.0 - 4.4	2	4.2	8.4
4.5 - 4.9	10	4.7	47
5.0 - 5.4	25	5.2	130
5.4 - 5.9	15	5.7	85.5
6.0 - 6.4	8	6.2	49.6
6.5 - 6.9	5	6.7	33.5
7.0 - 7.4	4	7.2	28.8
7.4 - 7.9	1	7.7	7.7
Total (f)	70	**Total (f × x)**	390.5

1. Calculate and write the missing data on the table.

2. What is the mean of the data? *Multiplying the mid points by the frequency gives the total of all the measurements (390.5 cm)*

   390.5 ÷ 70 = a mean of 5.6 cm

MEDICAL MATTERS

A doctor tries to keep each consultation to 20 minutes. To test whether she is keeping to schedule she asks her receptionist to record the time taken for the next 100 patients. The results of the survey are below.

Time (minutes)	14	15	16	17	18	19	20	21	22	23	24	25	26	27
Number of Patients	2	5	8	12	18	22	15	12	1	0	2	1	1	1

1. Give the range of times taken for consultations. *27 − 14 = 13 minutes*

2. Find the mode time taken per patient. *19 minutes*

3. Find the median time taken per patient. *19 minutes*

   Median data position (100+1)÷2 = 51.5

4. Find the mean time taken per patient.
   (14×2)+(15×5)+(16×8)+(17×12)+(18×18)+(19×22)+ ..... ÷100

   Total is 1877 ÷ 100 = 18.77 minutes (18 min 46 seconds)

5. On average, is the doctor managing to keep to her 20 minutes per consultation? Explain your answer.
   The median and mean suggest that she is keeping to time.

BLACK Worksheet 35

STATISTICS

An ordered set of data values (golf scores) is listed below:

69   71   73   *a*   78   98   *b*

1.   The data has a median of 74 and a mean of 82. What are the values of *a* and *b*?

a is the median 74 (it is the middle number)

mean 82 = (69 + 71 + 73 + 74 + 78 + 98 + b) ÷ 7

82 × 7 = 463 + b

b = 111

2.   In the case above, which would be the most reliable "average" of the numbers - the median or the mean. Give a reason for your answer.

In the case above the best average would be the median. This is because 5 out the 7 numbers are between 69 and 78.

3.   Study the stem and leaf graph below and give the minimum, maximum, mode, lower quartile, median and upper quartile. Use these figures to draw a box and whisker plot.

2	3 3 4
2	6 7 8 9 9
3	0 1 1 2 3 3 3 4
3	5 6 7 8 8 8 9
4	1 2 2 4
4	5 7 9
5	2 4
5	8
6	0

minimum = 23

maximum = 60

mode = 33

lower quartile = 29.5

median = 35

upper quartile = 43
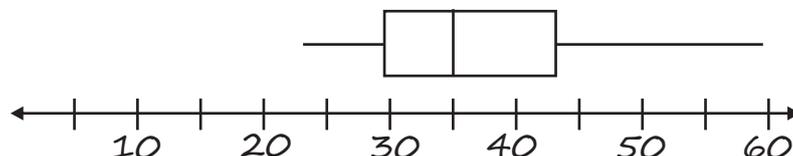
Knowing Transformation Geometry . . . . . complete these sentences.

An axis of symmetry is a line through which one side of the figure can be __*reflected*__ onto the other. The order of symmetry of a figure is the number of __*lines of symmetry*__ that it has.

When reflected, a point and its image are __*equal*__ distances from the mirror line. Length, angle size, and area are __*invariant*__ (stay the same), however sense can change (the way the figure looks).

In any transformation, the original points are given letter names e.g A, B and C while their images are indicated by __*A´*__ , __*B´*__ and __*C´*__

When a figure can be rotated onto itself one or more times during one complete turn it is said to have __*rotational symmetry*__ To rotate a figure you must know the centre of rotation and the angle of rotation. Positive angles of rotation move a figure __*anti-clockwise*__

When a figure is enlarged you must have the scale factor (k) and the centre of enlargement. If k > 1 then the figure is __*larger*__ than the original. If 0 < k < 1 then the figure is __*smaller*__ than the original. If k is negative, then the image is reversed to the opposite side of the centre. To find the centre of enlargement, join two or more related points of the original and image. The centre of enlargement is found where the lines __*intersect*__ To find the scale factor of an enlargement, divide the length of an image line by the corresponding length of the original.

A translation is defined by a vector. All points of the original are moved according to the vector.
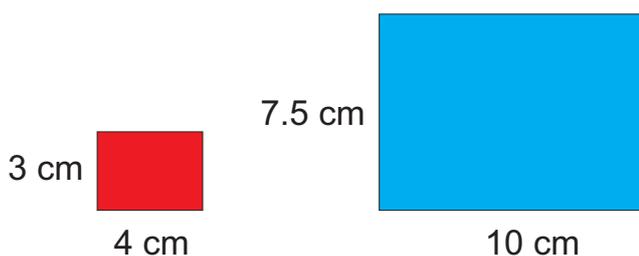
ENLARGEMENT BY RATIO

7.5 cm

3 cm

4 cm

10 cm

The two rectangles are similar.
What is the ratio of their measurements?

__*2: 5*__

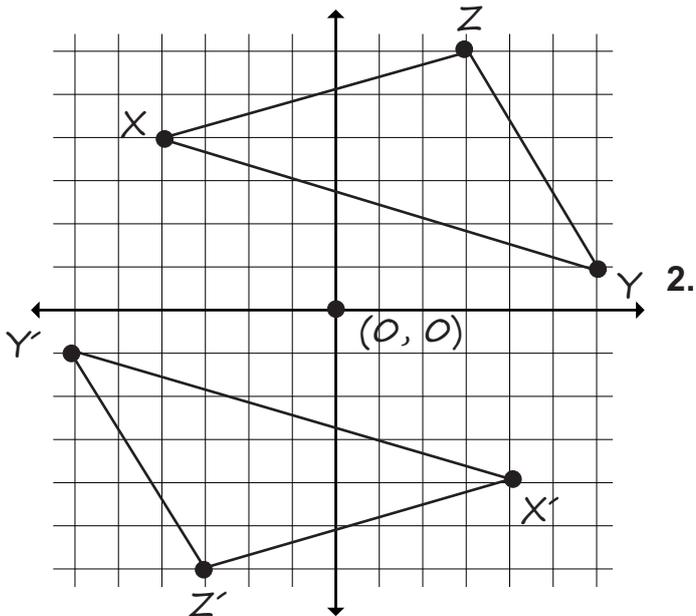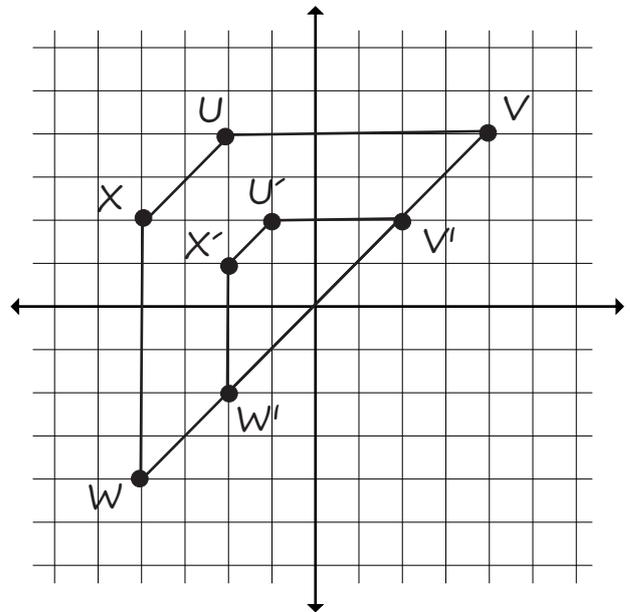## TRANSFORMATIONS

Draw the following figures and their images.

1. Draw trapeziod UVWX.
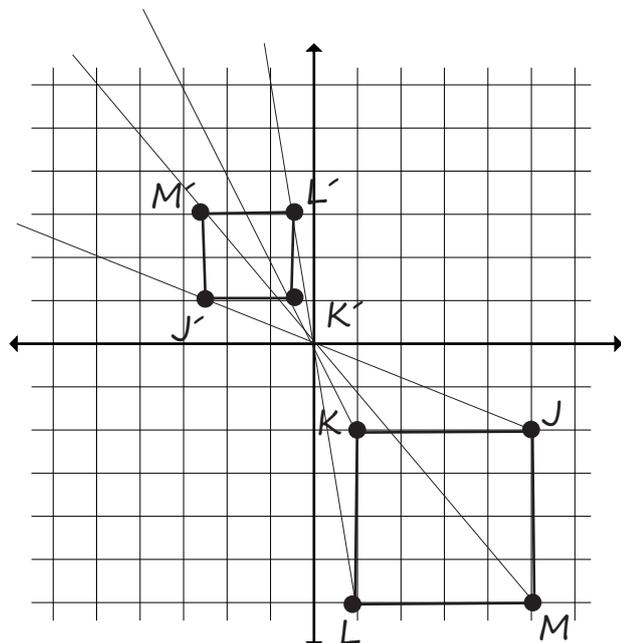U(-2, 4), V(4, 4), W(-4, -4), X(-4, 2)

Enlarge UVWX by scale factor ½
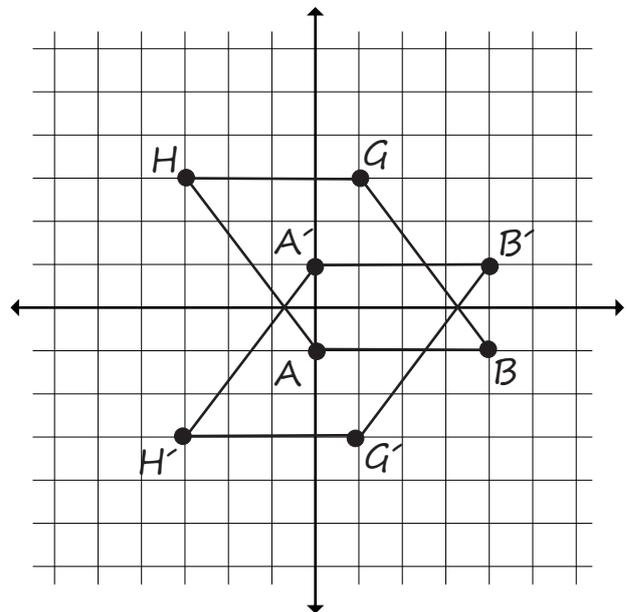Use centre of enlargement (0, 0).
Draw the image as U´V´W´X´

2. Draw triangle XYZ.
X(-4, 4), Y(6, 1), Z(3, 6)

Rotate XYZ 180° around (0, 0).

3. Draw parallelogram ABGH.
A(0, -1), B(4, -1), G(1, 3), H(-3, 3)

Reflect ABGH in the x axis.
Draw the image as A´B´G´H´.

4. Draw square JKLM
J(5,-2), K(1, -2), L(1, -6), M(5, -6)

Enlarge JKLM by scale factor -½
Use centre of enlargement (0,0)

MATHEMATICS AND HEALTH APPLICATIONS

1. A normal person's body weight is approximately 2 kg of muscle for each 5 kg of body weight. Make a table to show the relationship between body and muscle weight for people weighing between 60 and 110kg. Graph your relationship.

For each weight, divide by 5 then multiply the result by 2

as in the ratio each 2 parts out of 5 is muscle.

Body Weight:	65	70	75	80	85	90	95	100	105	110
Muscle Weight:	26	28	30	32	34	36	38	40	42	44

Graph of Body Weight v Muscle Weight
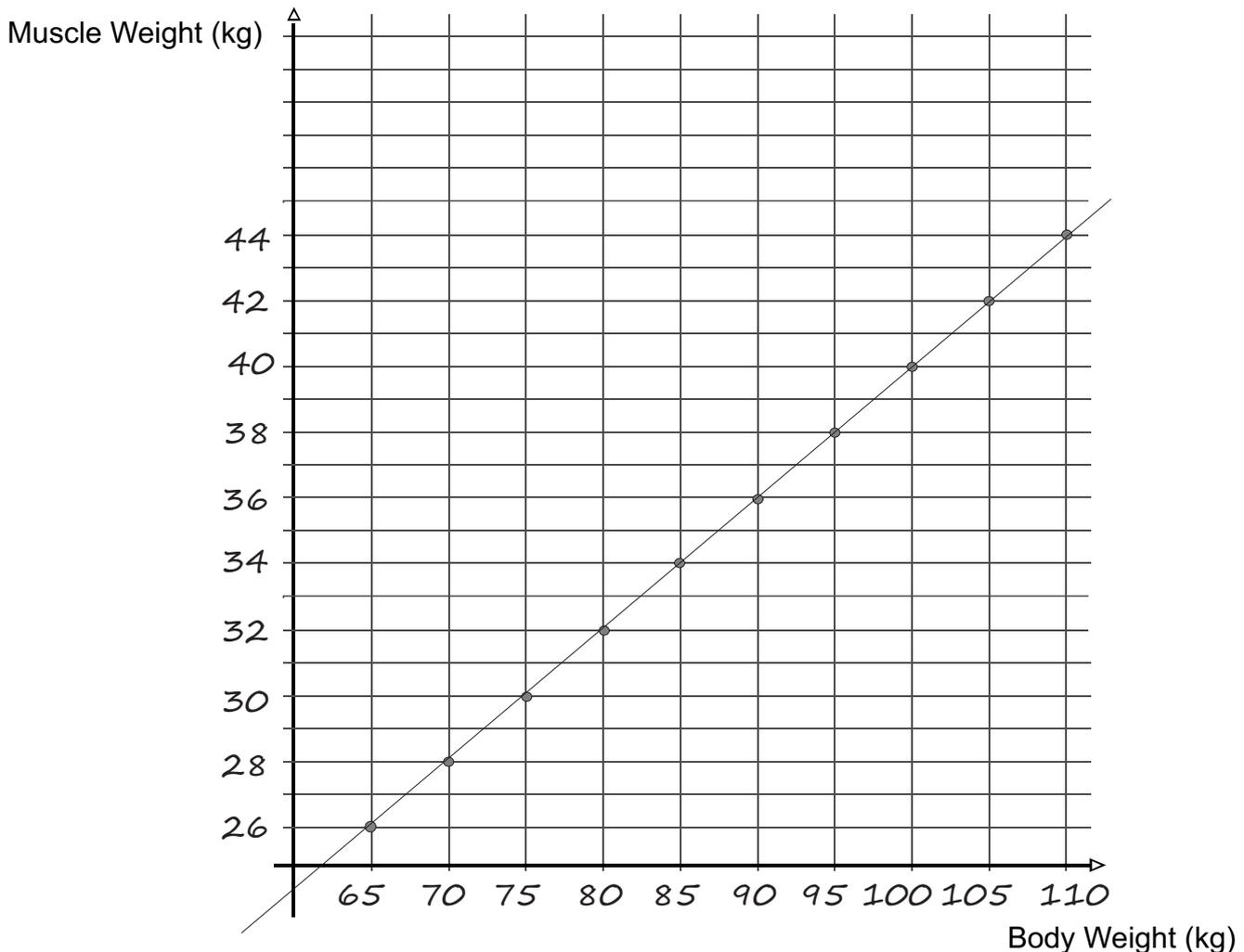
2. Nurses setting up intravenous (IV) fluids must control the flow rate (F) in drops per minute. They use the formula: $F = \dfrac{Vd}{t}$ where V is the volume of the solution in millilitres, d is the drop factor in drops per millilitre, and d is the time in minutes. Write the equation and calculate the correct flow rate of 1200 milliltres of saline to be given over 8 hours using a drop factor of 10 drops per millilitre.

$$F = \frac{1200 \times 10}{8 \times 60}$$

F = 25 drops per minute

BLACK Worksheet 36

SEQUENCES

The nth term of some sequences are listed below.
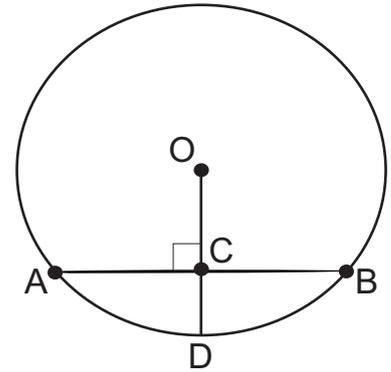Write the first five terms of each sequence.

1. $n(n + 2)$ *3, 8, 15, 24, 35*

2. $100 - 10n$ *90, 80, 70, 60, 50*

3. $n^2 - 1$ *0, 3, 8, 15, 24*

4. $n^3 + 1$ *2, 9, 28, 65, 126*

5. $\dfrac{n}{n + 1}$ $\dfrac{1}{2}, \dfrac{2}{3}, \dfrac{3}{4}, \dfrac{4}{5}, \dfrac{5}{6}$

6. $\dfrac{n}{2n + 1}$ $\dfrac{1}{3}, \dfrac{2}{5}, \dfrac{3}{7}, \dfrac{4}{9}, \dfrac{5}{11}$

PROBLEM SOLVING

In the circle above:

AB = 24 cm
OD = 13 cm

What is the length of OC?

OA = 13 cm (radius)

AC = 12 cm (½ AB)

Using Pythagoras

$13^2 = 12^2 + OC^2$

$OC^2 = 13^2 - 12^2$

$OC^2 = 25$

OC = 5 cm

MELENCOLIA

The famous ingraving entitled Melencolia has many mathematical instruments featured, including a magic square. It was drawn by the German artist Albrecht Durer.

Investigate the engraving on the internet, show where it is on the picture and write the numbers from the magic square in the grid below. Circle the numbers on the magic square which indicate the year in which the engraving was produced and write why it is a magic square.
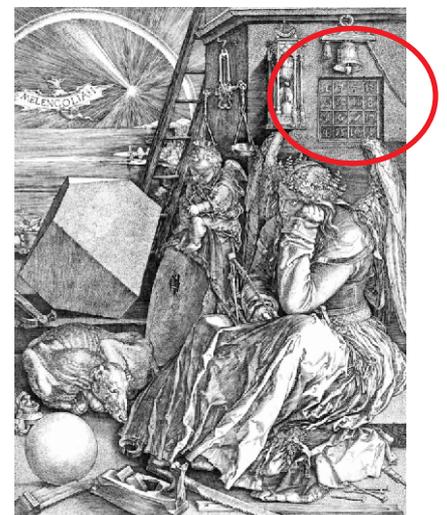
Columns, rows and diagonals all sum to 34.

16	3	2	13
5	10	11	8
9	6	7	12
4	15	14	1

Date of the engraving = 1514

Knowing Graphs . . . . . complete these sentences.

A linear graph is a straight line graph. Co-ordinates are number pairs that indicate points on the graph. The first number of the pair gives the horizontal _____*x value*_____ while the second number gives the vertical y value. Intercepts are where the line intersects the x axis _____*y intercept*_____ (x intercept) or the y axis (*y intercept*). The equation of any straight line can be written in the form y = mx + c, where m defines the _____*gradient*_____ of the line and c is the

**Litres of Water
Used in a Shower**

Litres of Water

Minutes Spent in Shower
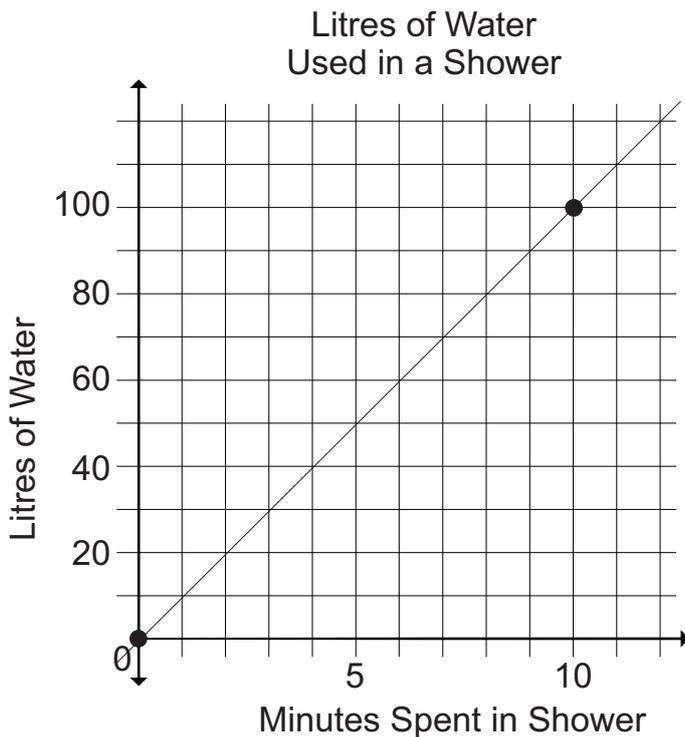
The number of litres of water used in a shower depends on the amount of time spent in the shower.

Look at the graph on the left and write a sentence that defines the equation of the graph.

The shower is using 10 litres

every minute. Therefore the

equation is L = 10m

where L = litres of water used

m = time spent in the shower

(in minutes)

Mark the following aspects onto the graph shown on the right.

    Positive gradient

    Negative gradient

    Origin

    y intercept

    x intercept

    Intersection of two line graphs.

Intersection of two line graphs.

Y intercepts

positive gradient

Negative gradient

Origin

X intercepts
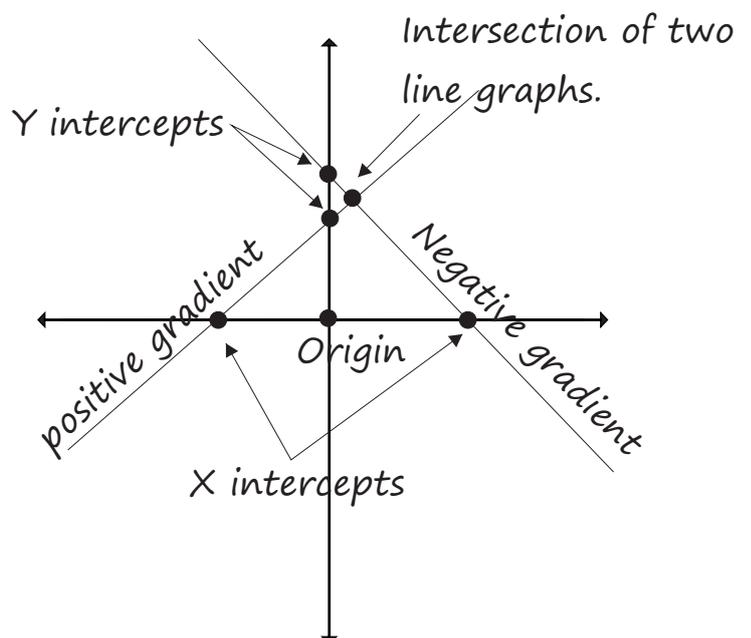
STRIKING STATISTICS

Edith and John decide to join their local Ten Pin Bowling Club. In his first 12 games, John's scores are:

160, 150, 141, 144, 138, 148, 140, 160, 158, 144, 141, and 140

The statistics for these twelve games are: Mean Score, 147; Median Score, 144; Upper Quartile Score, 158; Lower Quartile Score, 140; Range, 22.
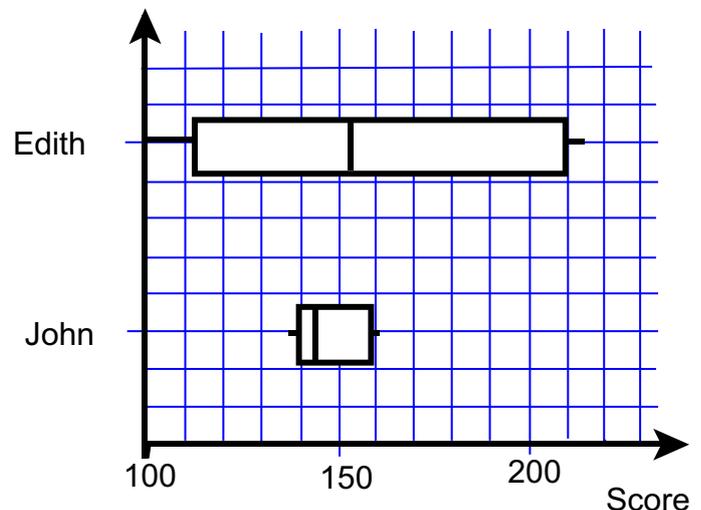
Edith's scores for her first twelve games are: 210, 215, 180, 131, 157, 147, 105, 112, 210, 205, 112 and 100

Scores in numerical order are

100, 105, 112, 112, 131, 147, 157, 180, 205, 210, 210, 215.

1. Summarise Edith's results in the table below.

1884 ÷ 12

Mean Score   *= 157*

Median Score   *152*

Upper Quartile Score *210*

Lower Quartile Score *112*

Range   *115*

2. On the grid above, draw two box and whisker graphs: one that shows John's statistics and the other that shows Edith's statistics.

3. There is a ten pin bowling doubles competition in the next few weeks. If you were to enter and needed a partner, would you choose Edith or John? Write an explanation below about why you would choose that particular person.

If you choose Edith – although she has very high scores, she also has low ones. Gamble on her having a "hot" game when playing with you.

If you choose John – he is a very consistent player with a narrow range of scores.

4. In his next game, John increases his mean score to 148. How many points did he score in the game to achieve this result?

148 = $\frac{(147 \times 12) + a}{13}$ therefore a (the score) = 160
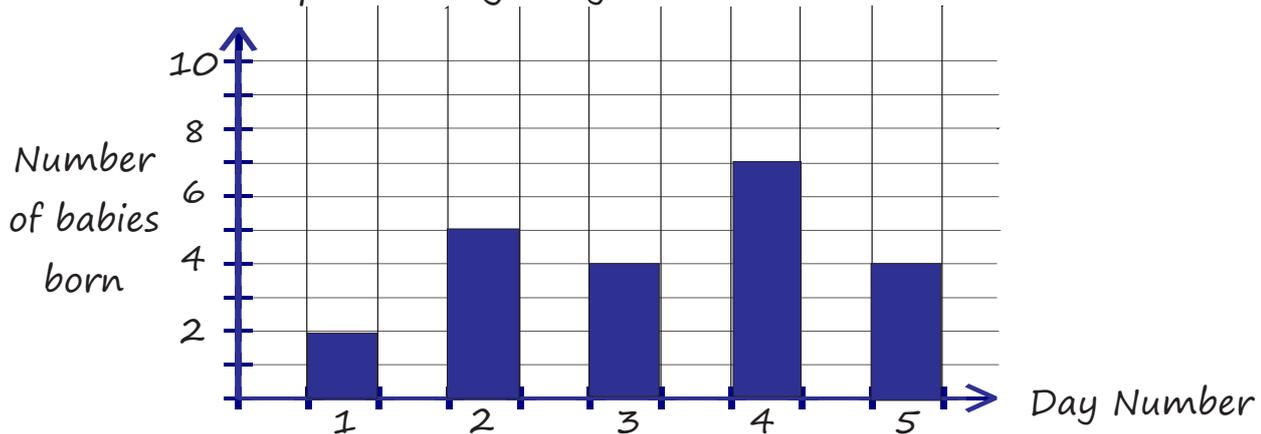
or use (13 x 148) – (12 x 147) = 160

MATERNITY MATHEMATICS

At Mahobe Maternity Hospital, babies are born each day. The table below shows how many babies were born over a particular 5 day period.

|	Day 1	Day 2	Day 3	Day 4	Day 5
Number of babies born	2	5	4	7	4

1. Draw and label a bar graph of the number of babies born each day:

Graph Showing Daily Number of Babies Born

2. On how many days were there more than 3 babies born? _____*4 days*_____

3. How many babies were born over the 5 day period? _____*22*_____

4. Calculate the mean number of babies born per day. _____*22÷5 = 4.4*_____

5. What was the most common number of babies born per day (the mode)? _*4*_

This next two diagrams show the probability of a couple's first child being a boy or a girl and the probabilities for the first two children.

The First Child

The First Two Children
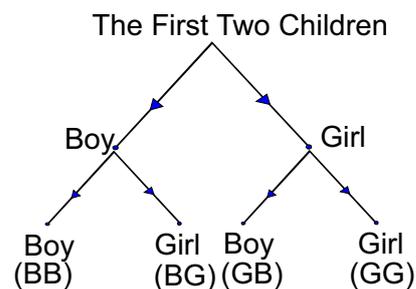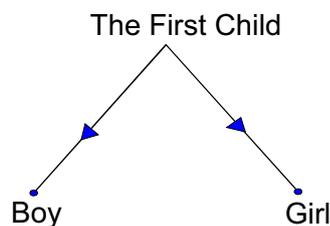
6. What is the probability of a family of two children being 2 boys?

0.25

7. What is the probability of a family of two children being a boy and a girl?

(BG + GB) = 0.5

8. What is the probability of a family of three children being 2 girls and 1 boy?

2 girls and 1 boy = (BGG, GBG, GGB), $3 \times (\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}) = \frac{3}{8}$

BLACK Worksheet 37

PREDICTING EXAMINATION MARKS

Teachers quite often need to predict marks when students are absent on the day of test. They do this by looking at results from previous tests. The table below gives ten marks from students who have both sat the same two Mathematics tests.

Result 1	47	74	69	79	64	37	63	89	53	45
Result 2	35	53	50	59	48	26	44	65	38	32

1. Plot these results on a scatter diagram and draw a line of best fit.

Scatter Plot of Test 1 Results V Test 2 Results

(85, 63)

(60, 44)

(20, 14)

Mathematics Test 2 Result

Mathematics Test 1 Result
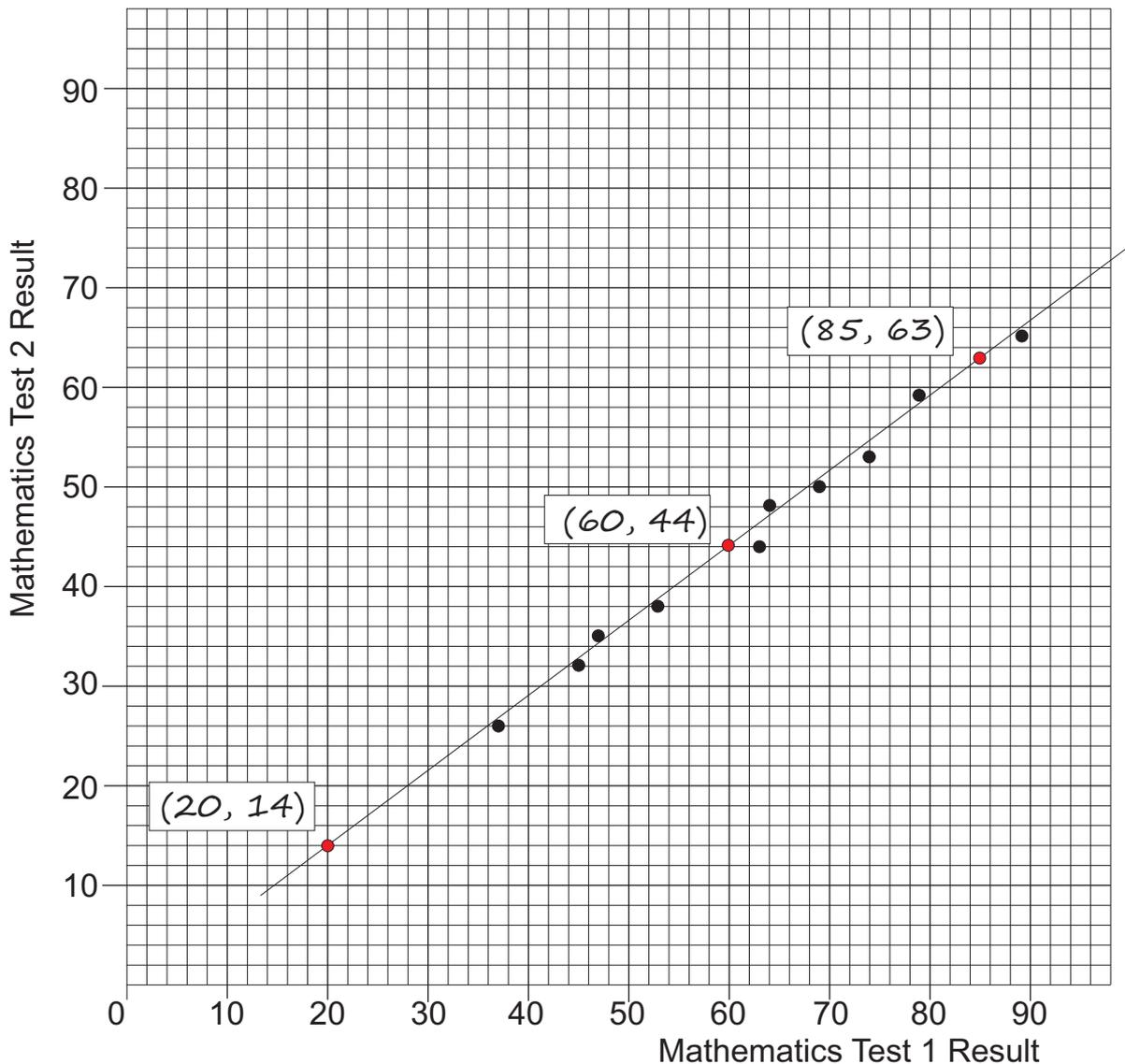
2. Three students missed Test 2. Their marks for Test 1 were 60, 85 and 20. What would be a good estimate of these three students marks for Test 2? *See graph*

TANGENT TRIGONOMETRY

1. Clothespin is a famous stainless steel sculpture found in Centre Plaza Philadelphia, USA. At a certain time of the day you measure the shadow at 35.0 metres from the base and an angle of 32º (see the diagram below). Use these measurements to calculate the height of the sculpture.

$$\tan 32° = \frac{height}{35}$$

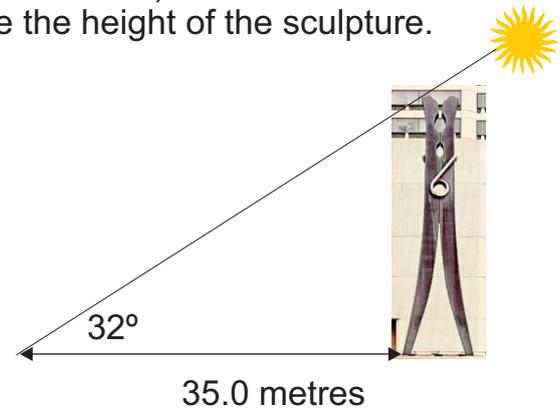$$\tan 32° \times 35\ m = 21.87\ m$$

Clothespin is 21.9 m tall

32º

35.0 metres

2. Clavius is one of the largest crater formations on the Moon (see the photo below). Astronomers have calculated that when the sun's rays strike the moon's surface at an angle of 0.9º, the edge of the lunar crater casts a shadow along the whole length of the crater. They know that the Clavius crater is 225 km wide, but what is its depth?

sun's rays

depth

0.9º
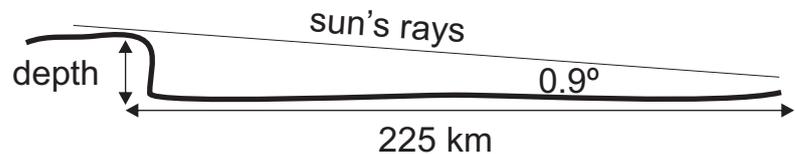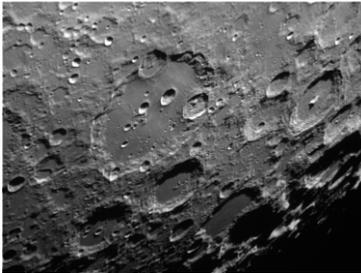
225 km

$$\tan 0.9° = \frac{depth}{225\ km}$$

$$\tan 0.9° \times 225\ km = 3.53\ km$$

The Clavius Crater is about 3.53 km deep

3. A water tower in Stanton, Iowa is recognised as the world's "largest coffeepot". Two angle measurements are taken 15 metres from the base of the structure. The angle of elevation from the ground to the top of the pot is 68º, the angle of elevation from the ground to the bottom of the pot is 61º. How tall is the actual coffee pot?

$$\tan 68° = \frac{height}{15\ m}$$

$$\tan 68° \times 15\ m = 37.1\ m$$

$$37.1 - 27.1 = 10\ metres$$

The coffee pot must be 10 m in height

$$\tan 61° = \frac{height}{15\ m}$$

$$\tan 61° \times 15\ m = 27.1\ m$$

EXTENDED PROBLEM SOLVING

For the period 2000 - 2010 the consumption of bottled water in New Zealand has been approximated by $y = 0.1x^2 + 0.1x + 6$ where y is the volume of water in litres per person and x is the number of years since 2000.

1.   Either use a spreadsheet or a calculator to find the values and fill in the table.

Assume that the year 2000 = year 0

Year (x)	1 2001	2	3	4	5	6	7	8	9	10
$0.1x^2 + 0.1x + 6$	6.2	6.6	7.2	8	9	10.2	11.6	13.2	15	17

2.   What was the first year when consumption of bottled water went over an average of 10 litres per person?

2006

3.   Graph the results below. Between which years did the annual water consumption start to increase the most dramatically from the previous year?
Explain how you used the graph to find this.
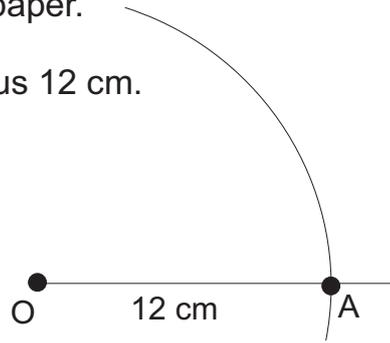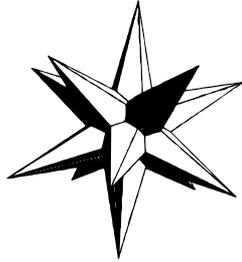
Graph of Average Water Consumption Since 2000
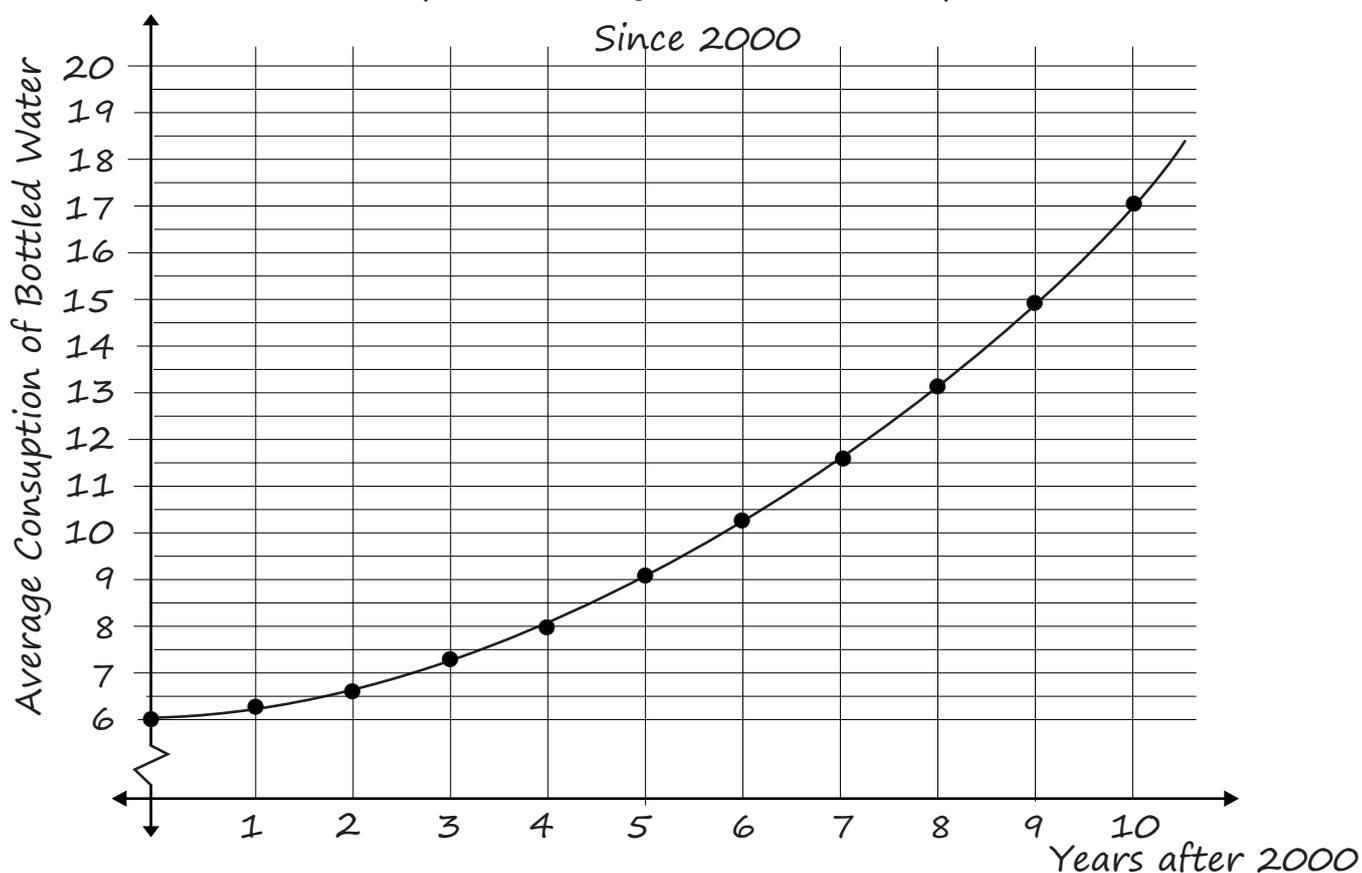
*After year 9 (2009) the graph begins to get the steepest*

THE NINTH STELLATION OF THE ICOSAHEDRON

This is a solid figure made from 12 star points. To make each start point you need to draw the pattern below on 12 pieces of stiff card or paper.
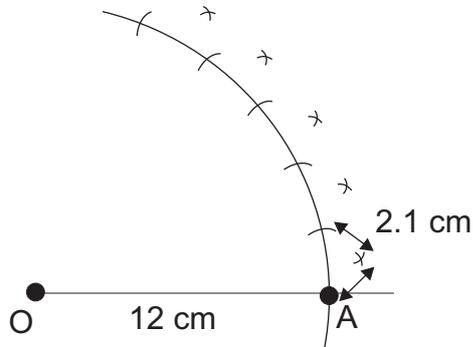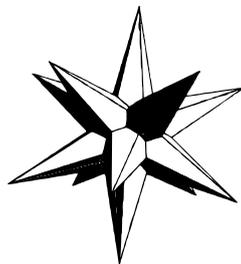
**Step 1:** With centre O, draw an arc with radius 12 cm.

Step 2: Beginning at point A draw an arc with radius 3.3 cm.
Continue until you have 5 points on your arc.
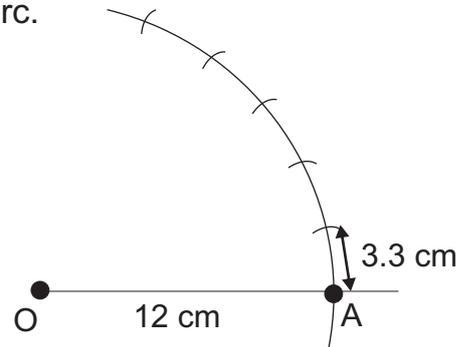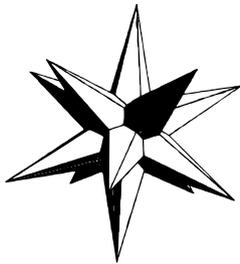
Step 3: Using A and the other points on the arc mark off the points at 2.1 cm

Step 4: Draw all the lines and add tabs as shown. Score along the lines and fold. Glue the longest tab to the opposite face and you should have a star point. Remember you need 12 of these. Each star point is glued to another via two of the small outside tabs
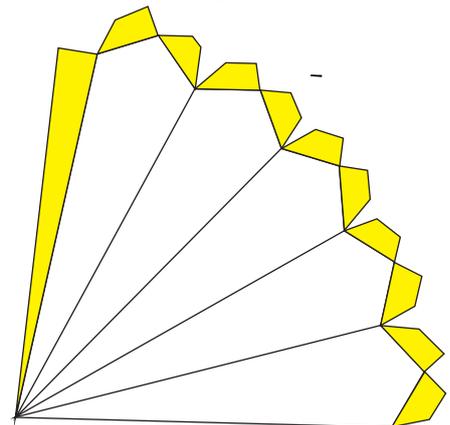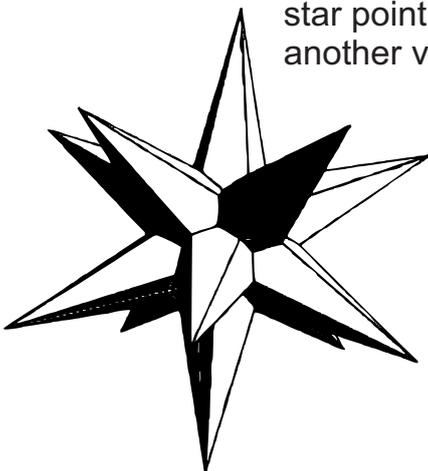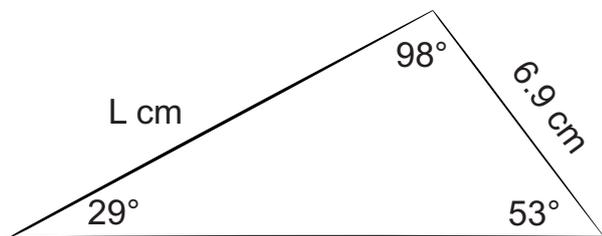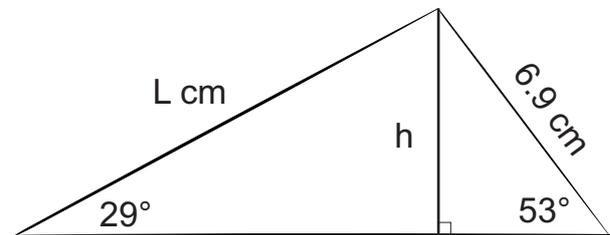
BLACK Worksheet 38

ADVANCED TRIGONOMETRY

Trigonometry can be used to work out the angles and sizes of right angle and non right angled triangles. The triangle below does not have a right angle however in the Step 2 illustration two right angled triangles are created.

1. Study the triangles below then complete the calculations that have been started to find the size of the triangle side labeled L cm.

Step 1 — 98°, L cm, 29°, 53°, 6.9 cm

Step 2 — L cm, h, 29°, 53°, 6.9 cm

$$\frac{h}{L} = \text{Sin } 29$$

$$h = L \times \text{Sin } 29$$

$$\frac{h}{6.9} = \text{Sin } 53$$

$$h = 6.9 \times \text{Sin } 53$$

$$h = h$$

therefore $L \times \text{Sin } 29 = 6.9 \times \text{Sin } 53$

$$L = \frac{6.9 \times \text{Sin } 53}{\text{Sin } 29}$$

$$L = 11.4 \text{ cm}$$

2. What is the perimeter of the square ABCD?

A, B, C, D, 10 cm
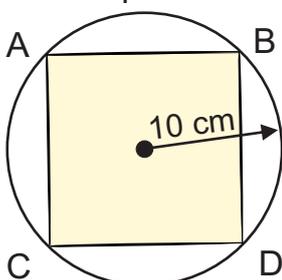
The diagonal of the square is 20 cm

If each side is x cm then using Pythagorus

$x^2 + x^2 = 20^2$ and $x = 14.1$ cm

Therefore the perimeter = 56.4 cm

USING TRIGONOMETRY

1.   A forester measures 50 meters from a group of trees and uses a clinometer to measures the angle of elevation from eye level to the top of the trees at 49.5°. The distance from the forester's eye level to the ground is 1.5 meters.
Find the height of the tree to the nearest tenth of a meter.

$\tan 49.5° = \dfrac{height}{50}$

$\tan 49.5° × 50\ m = 58.54\ m$

Tree is 1.5m + 58.5 m = 60

Height is 60.0 metres (nearest tenth of a metre)

1.5 m

49.5°
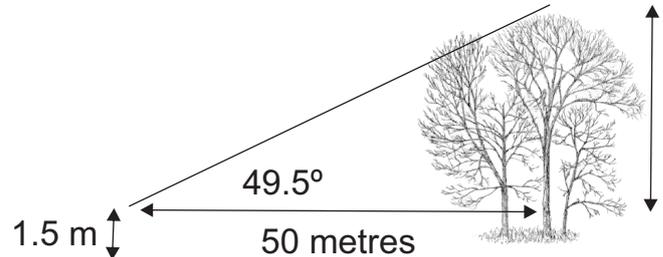
50 metres

2.   The largest sundial in the world was built around 1720 at Jaipur, India. It is a right angled triangle shape and the shadow cast by the sun visibly moves 1mm every second.

If the hypotenuse length of the sundial is 60.35 metres and the angle of elevation is 27°, find the height of the sundial from the ground.

$\sin 27° = \dfrac{height}{60.35}$

$\sin 27° × 60.35\ m = 27.398\ m$

Height is 27.4 metres (nearest tenth of a metre)

3.   At the beginning of a Synchronized Swimming performance, two swimmers start at opposite corners of a rectangular pool that is 50 meters long and 25 meters wide. They swim toward each other along a diagonal and meet halfway. How far from their starting points do the swimmers meet?

$Diagonal^2 = 50^2 + 25^2$

$Diagonal^2 = 3125$

Diagonal = 55.90 m therefore ½ diagonal = 27.95 m

4.   A building's wheel chair access ramp cannot be over 5° in elevation. You are to build a ramp that is 4 metres long. How far away from the building should the ramp start?

$Cos\ 5° = \dfrac{d}{4}$

$Cos\ 5° × 4\ m = 3.985\ m$

Distance should be between 3.985 and 3.999 metres

4 metres

5°

distance from building (d)
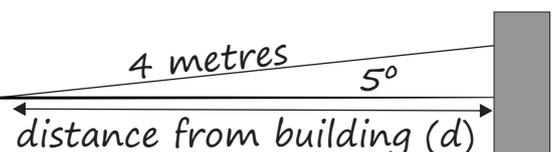
The formula: $s = \frac{1}{6} n (n + 1)(2n + 1)$ gives the sum of the squares of the integers from 1 to n. For example, the sum of the squares of the integers from 1 to 4 $(1^2+2^2+3^2+4^2)$ is

$$\frac{1}{6} (4) \times (4 + 1) \times ((2\times4) + 1)$$

$$= \frac{1}{6} \times 4 \times 5 \times 9$$

$$= 30$$

Use the formula to find s when n = 10.

$$= \frac{10}{6} \times (10 + 1)(20 + 1)$$

$$= \frac{10}{6} \times 231$$

$$= 385$$

Police use the formula $S = \sqrt{204D}$ to estimate the speed (S) of a car in kilometers per hour by measuring the distance (D) that a car skids on a dry road.

On his way home from work, David skids while stopping to avoid a dog on the road and crashes into a parked car. He tells the police that before the accident he was only traveling at 50 km per hour. The police measure the skid at 13.3 metres. Should David be prosecuted for speeding if the road's speed limit was 50 km/hr?

$$S = \sqrt{204 \times 13.3}$$

$S = 52.1$ km / hour   David was probably traveling at a safe speed before the accident so therefore should not be prosecuted.

The gold content of jewelry is measured in Karats (abbreviated to K or KT). A Karat is the unit of proportion based on a range of 1 to 24 parts, with 24 karat gold being 100% pure. However because pure gold is too soft for ordinary use it is typically alloyed with other metals such as copper, silver and nickel to increase the hardness.

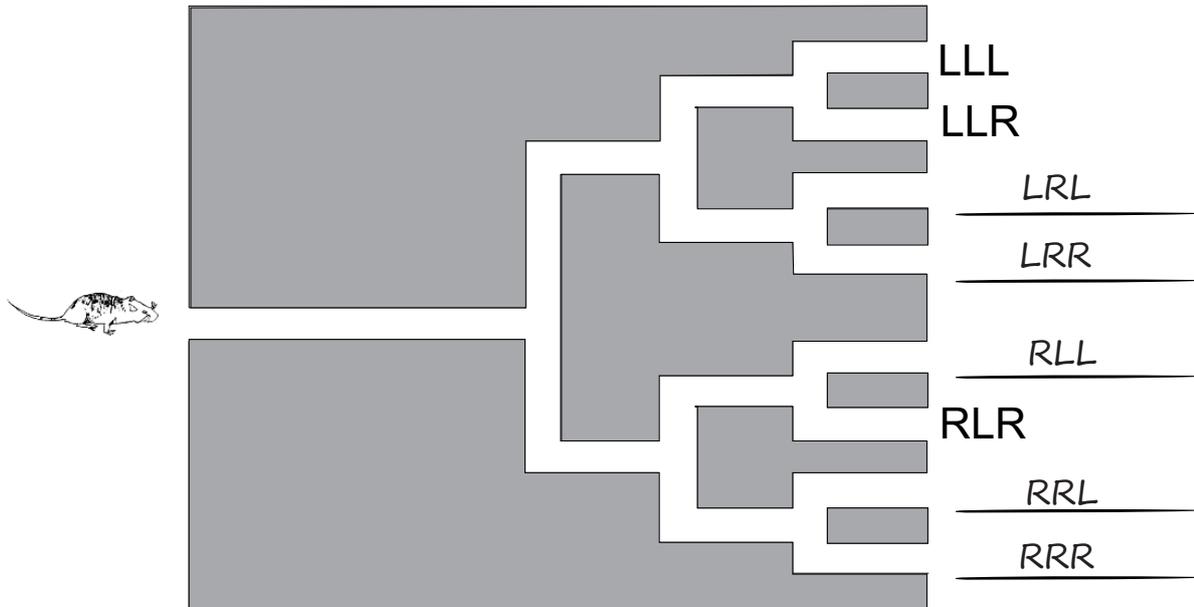This means 18 Karat gold is 75% pure gold ($\frac{18}{24}$).

How would you describe a piece of jewelry which is $\frac{2}{3}$ pure gold using the unit Karats?

Convert $\frac{2}{3}$ to an equivalent fraction over 24

$$= \frac{16}{24}$$

This means the jewelry is 16 Karats

The figure above shows the view looking down on a maze. Mice enter the maze from the left hand side and at each branch they can turn either right (R) or left (L).
Each exit can be labelled according to the number of left or right turns that were made.
For example RLR means the mouse turned right at the first branch, left at the second branch and right at the third branch.

1. Label each of the unmarked exit points.

2. How many of the exits have exactly two L's in their label? _____ 3 _____

3. What fraction of the exits have only one L in their label? _____ $\frac{3}{8}$ _____

4. 64 mice go through the maze.
How many would you expect to follow the path to the exit labelled LLL?

    64 ÷ 8 (different exits) = 8 mice

INTERPRETING A MARGIN OF ERROR

It has been found that when you survey different samples from the same population you get can get different results. For example one survey held at your school might find that 68% of students have cell phones. Another survey might find that 72% have cell phones. This is because the surveys don't ask the whole school but might only take a sample of 100 students each time. Due to this variation a survey should include a "margin of error". This means a sample percent of 30% with a margin or error of ±5% means that the actual percent is most likely to be between 25% and 35%.

In an election a random sample predicts that Arnold will receive 52% of the votes and Xania will receive 48% of the votes. The margin of error is ±3%. Can you predict who will win the election?

Arnold: 52% – 3% = 49%, 52% + 3% = 55%, i.e. between 49% – 55%

Xania: 48% – 3% = 45%, 48% + 3% = 51%, i.e. between 45% – 51%

The intervals overlap so you cannot confidently predict who will win.

BLACK Worksheet 39

HOLIDAY PROBLEMS TO PONDER

1. Your neighbours tell you that they are going on holiday. They will be away for "43 meals". How many days is that?

   43 ÷ 3 = 14 days with 2 remainder. (Assume 3 meals a day.)

   They will be away for 15 days (the 2 remainder still indicated meals)

2. Last Christmas, motorists in Germany were held up on the autobahn linking Frankfurt and Munich by heavy snow storms. The traffic jam was 170 km long. How many cars do you estimate were in the traffic jam? Write the assumptions you made with your estimate.

   If a typical car is 5 metres long then 170 000 ÷ 5 = 34,000 cars.

   However the question does need more information – were there 1, 2 or 3 lanes of traffic; were the cars "bumper to bumper"? The figure doesn't take into account other forms of traffic such as motor bikes or trucks.

3. A report in the newspaper says that the average New Zealand male will change the TV channel at least 117,000 times in their lifetime. Is this a realistic figure?

   The average life span of a male is 75 years.

   117000÷75 = 1560 channel changes per year.

   1560 ÷ 365 (days) = 4.27 times a day.
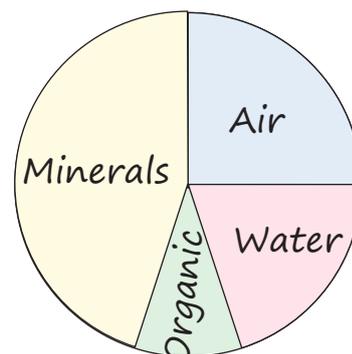
   Is this realistic for you?

GRAPHICAL REPRESENTATIONS

1. A soil sample was taken from a site in Newmarket, Auckland. The composition of the soil was as follows:
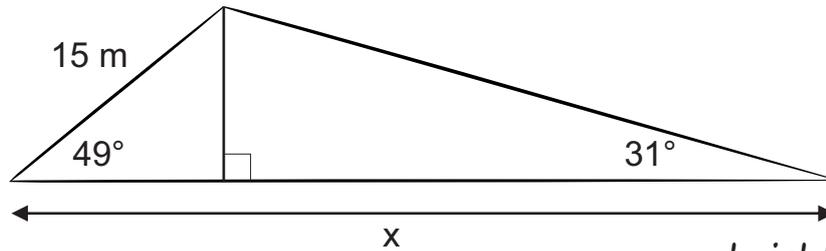
   Air 25%   *0.25 × 360 = 90°*
   Water 20%   *0.2 × 360 = 72°*
   Minerals 45%   *0.45 × 360 = 162°*
   Organic Material 10% *0.1 × 360 = 36°*

   Draw a pie chart to represent the figures.

MATHEMATICAL CHALLENGES

1. Study the triangle below and calculate the distance x

First find the height of the triangle $\sin 49° = \dfrac{height}{15}$

$\sin 49 \times 15 = 11.32$ metres

Length of the smaller triangle $\cos 49° = \dfrac{length}{15}$

$\cos 49° \times 15 = 11.32$ metres $= 9.84$ metres

length of the larger triangle $\tan 31° = \dfrac{11.32}{length}$

$length = \dfrac{11.32}{\tan 31°}$

$length = 18.84$

Total length (x) is $9.84 + 18.84 = 28.68$ metres

2. Calculate the area of the equilateral triangle below.
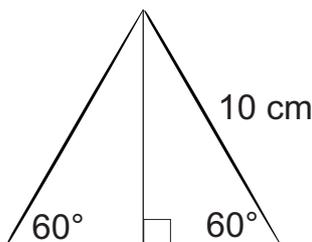
Calculate the height: $\sin 60° = \dfrac{height}{10}$

$\sin 60° \times 10 = 8.66$ cm

The base must be 10 cm (equilateral triangle)

Area $= ½ \times 10 \times 8.66$

$= 43.3$ cm²

3. A periscope is used in a submarine to view the surface of the ocean. The Naval Manual states that the periscope height, h, (in feet) above the surface and the distance, d, (in miles) that a sailor can see are related by the formula: $h = \dfrac{d²}{1.4}$

If the periscope is raised to a distance of 3 feet above the ocean surface, how far (in kilometres) can the sailor see?
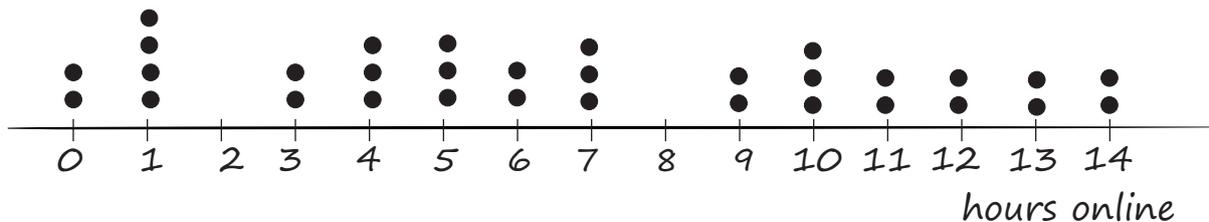
$3 = \dfrac{d²}{1.4}$ therefore $d² = 4.2$, d (in miles) $= 2.05$

convert miles to km $= 2.05 \times 1.61 = 3.30$ kilometres

STATISTICS

The list below gives the number of hours that students in one particular class spent online last week.

14 7   6   10 5   12 1   6   5   5   1   3   11 7   4   7
4   13 13 9   14 1   0   12 4   1   9   3   11 0   10 10

1. Make a line plot of the data.

hours online

2. Which value occurs the most frequently?

1 hour

3. Would the mean, median or mode best represent this data.
Explain your answer.

The mean (6.8) and the median (6.5) could both represent the data

accurately as they are both fairly central.

Look at the data in the stem and leaf plot below.

STEM	LEAF
9	3, 5
10	2, 2, 5, 8, 9
11	4, 5, 8, 8, 8, 8, 9, 9
12	0, 1, 7, 7, 8, 9 9	3 = 9.3

4. What is the difference between the least and the greatest values?

12.9 – 9.3 = 3.6

5. What is the mode of the numbers?

11.8 occurs the most (4 times)

6. What is the median of the numbers?

There are 21 numbers. Therefore the median is the 11th number (11.8)

7. Does the mean, median or mode best represent this set of numbers?

mean = 11.45, median = 11.8, mode = 11.8.

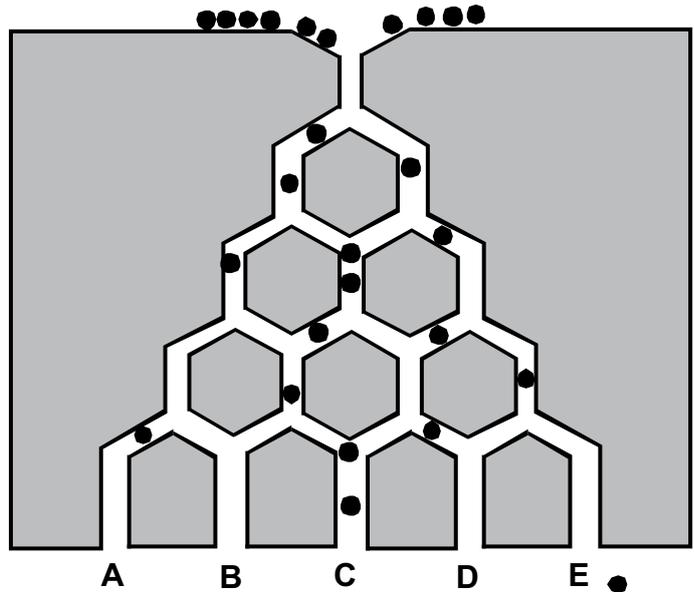*All are central and representative of the set of numbers.*

PROBABILITY EXPERIMENTS

Four groups of students each conduct an experiment with 100 marbles and the maze shown on the right.

The marbles are placed at the top entrance to the maze and allowed to travel downwards.

Their exit point is noted.

Each group's results are shown in the tables below.

A   B   C   D   E

Group 1

Exit	Number of marbles
A	10
B	26
C	35
D	24
E	5

Group 2

Exit	Number of marbles
A	5
B	27
C	37
D	20
E	11

Group 3

Exit	Number of marbles
A	6
B	25
C	40
D	22
E	7

Group 4

Exit	Number of marbles
A	4
B	24
C	38
D	28
E	6

1. Put together the results of each group and work out the probability of a marble arriving at each of the exits.

EXIT	A	B	C	D	E
Total number of marbles arriving at exit	25	102	150	94	29
Probability of marbles arriving at exit	0.063	0.255	0.375	0.235	0.073

2. At which exit or exits have the most marbles arrived?
Suggest a reason for this.

The most popular exit is C followed equally by B and D and then equally by A and E. There are more routes towards the exit C.

BLACK Worksheet 40

ALGEBRA ESSENTIALS

Simplify the following expressions:

1. $2 \times 3x$ $6x$

2. $3x \times 4y$ $12xy$

3. $2x \times 4y \times 5$ $40xy$

4. $4x \times -2y$ $-8xy$

5. $-2x \times -3y \times -4a$ $-24axy$

6. $-5 \times 3x \times 2y$ $-30xy$

7. $-3x \times 3x$ $-9x^2$

8. $2x^2 \times 5x^3$ $10x^5$

9. $4x \times -2x^2$ $-8x^3$

10. $-12x^3 \times -3x$ $36x^4$

Expand the following:

1. $(x + 6)(x + 9) =$ $x^2 + 15x + 54$

2. $(x - 7)(x - 10) =$ $x^2 - 17x + 70$

3. $(x + 15)(x - 3) =$ $x^2 + 12x - 45$

4. $(x - 4)^2 =$ $x^2 - 8x + 16$

5. $(x - 12)(x + 8) =$ $x^2 - 4x - 96$

6. $S = 4\P r^2$ and $\P = 3.14$ and $S = 80$
   Calculate the value of r

   Rearrange $r^2 = \dfrac{S}{4\P}$

   (using calculator) $r^2 = 6.37$

   Therefore r = 2.52 (2 dp)

FACTORISE THESE EXPRESSIONS

1. $x^2 + 5x + 6$ $x^2 - 7x - 30$ $x^2 - 16$

   $(x + 3)(x + 2)$ $(x - 10)(x + 3)$ $(x - 4)(x + 4)$

2. $x^2 + 7x$ $x^2 + 4x + 4$ $x^2 + 2 - 35$

   $x(x + 7)$ $(x + 2)^2$ $(x + 7)(x - 5)$

3. $x^2 - 14x + 48$ $2x^2 + 16x + 24$ $x^2 + 7x$
                   $= 2(x^2 + 8x + 12)$

   $(x - 8)(x - 6)$ $= 2(x + 6)(x + 2)$ $x(x + 7)$

STATISTICS AND REAL LIFE DATA

The table below gives the men's winning times in the Boston Marathon for every 10th year from 1900 to 2000. In the table the x value represents the number of years since 1900 and the y value represents the winning time (converted to the nearest minute)

x	0	10	20	30	40	50	60	70	80	90	100
y	160	149	150	155	148	153	141	131	132	128	130

1.   Make a scatter plot of each (x, y) pair and draw a line that appears to best fit through the data.

Boston Marathon Times Since (1900)
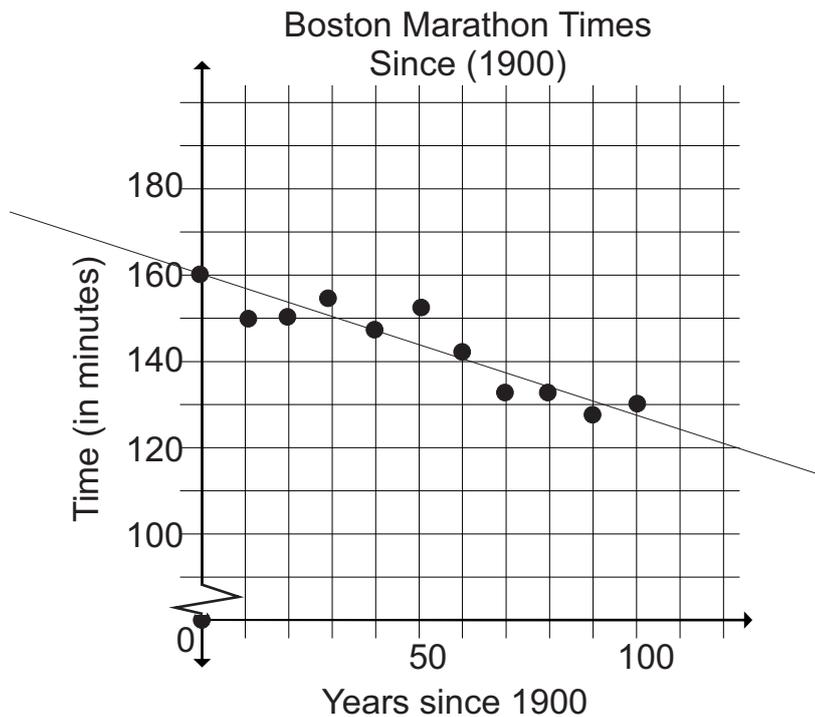
2.   Write an equation for your line.

The best-fitting lines will vary. Using (0, 160) and (90, 130)

the equation is y = –1/3x + 160

3.   Use your equation to predict the men's winning time (in minutes) for the Boston Marathon for 2010.

about 123 min (2 hours and 3 minutes)

4.   The actual winning time for the 2010 Boston Marathon was 2 hrs 5 min 52 sec (126 minutes.) Does the graph and resultant equation accurately predict winning times over each ten year block?

For the first 100 years it may be okay. If the line continues then

it predicts an eventual winning time of 0 minutes. This is impossible.

Notice how from 1970 the gradient through a best fit line is different.

Winning times may continue to drop but at a slower rate and will

eventually decrease at a much smaller rate.

ANIMAL AGES

The table below shows the average natural life span of 20 well known animals.

Animal	Life Span(Years)	Animal	Life Span (Years)
Baboon	20	Mouse	3
Camel	12	Opossum	1
Cow	15	Pig	10
Elephant	40	Rabbit	5
Fox	7	Sea Lion	12
Gorilla	20	Sheep	12
Hippopotamus	25	Squirrel	10
Kangaroo	7	Tiger	16
Lion	15	Wolf	5
Monkey	15	Zebra	15

1. Make a line plot of the average life spans of the animals in the table.
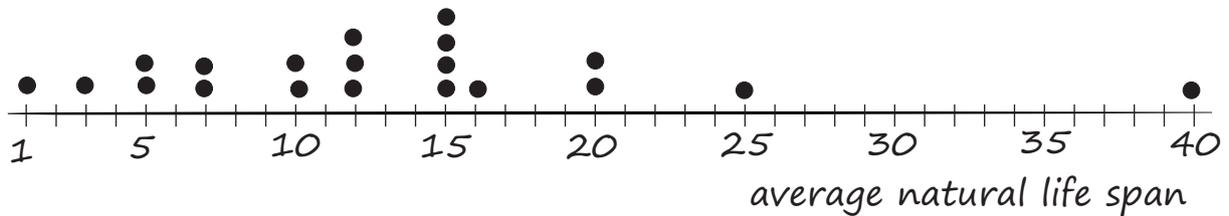
2. Write a statement about the average life span of animals.
Most of the animals in the group (over half) seem to live between 5-15 years. There are a couple that live 3 years or less and 5 that live over 15 years.

3. What is the most common life span for the animals in the table?
There are 4 animals with life spans of 15 years.

**4.** How many animals live at least 20 years?
There are 18 animals.

5. Is this table a good representation of age spans of all animals?
The table only has 20 animals so could not be considered a good representation of all animals. More data is needed to do an accurate assessment of animal life spans.
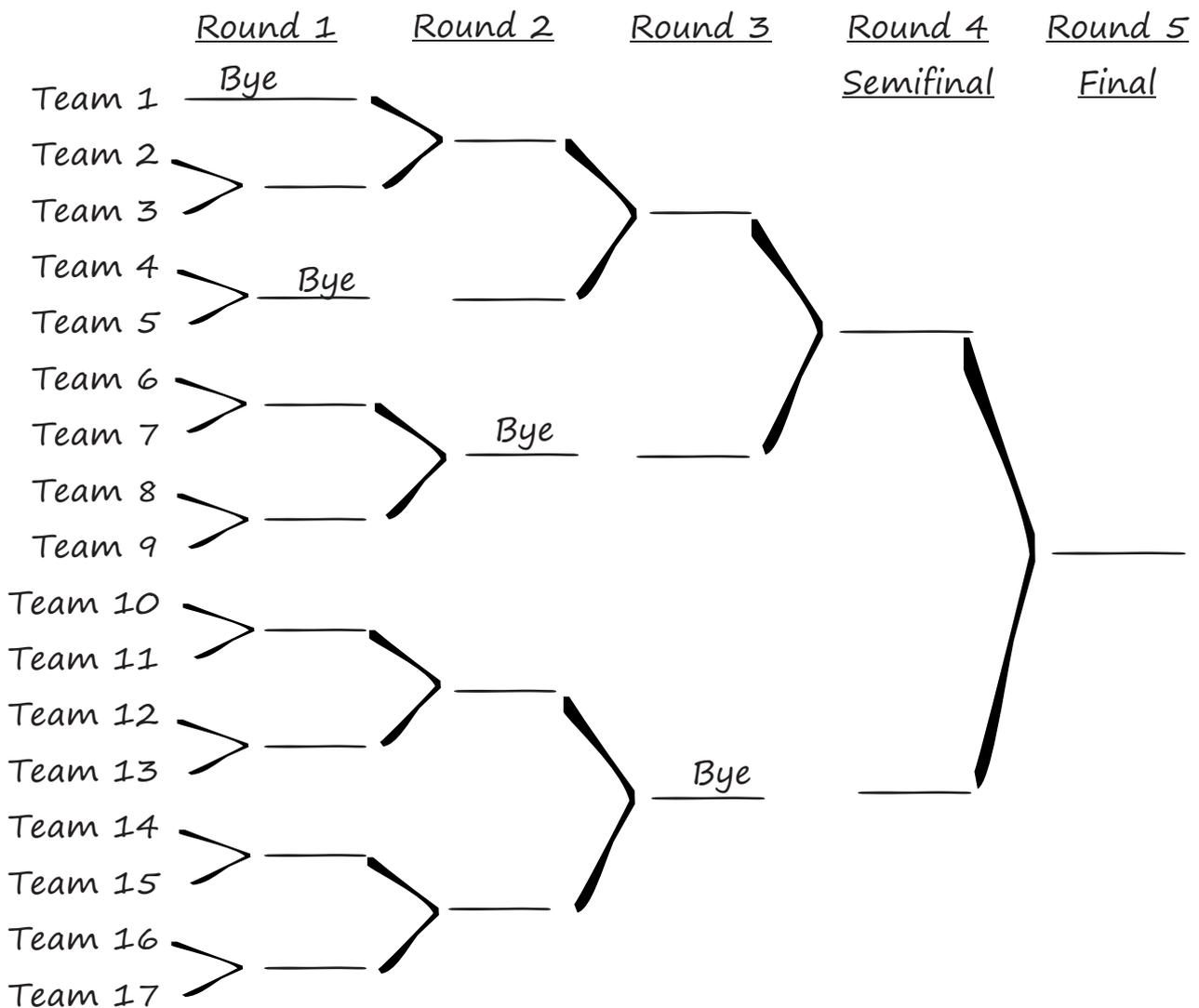
THE TENNIS DRAW

Mahobe College is holding an interschool tennis tournament. There are 17 teams entered in the tournament. The organisers decide to have an elimination tournament with each team drawn out of a hat for the first round of games. How many games and how many "byes" must be scheduled for the tournament?

The example below shows the schedule arrangement if there were 3 teams. In this case shown there would be 2 games played.

Team 1 ___Bye___ Team 1

Team 2

Team 3
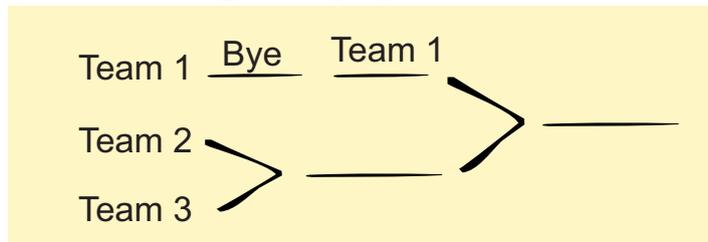
1. Draw a chart to show all games that could be scheduled.
2. How many games will be played with a 17 team tournament?

Round 1	Round 2	Round 3	Round 4 Semifinal	Round 5 Final

Team 1 ___Bye___

Team 2

Team 3

Team 4 ___Bye___

Team 5

Team 6

Team 7 ___Bye___

Team 8

Team 9

Team 10

Team 11

Team 12

Team 13 ___Bye___

Team 14

Team 15

Team 16

Team 17

There will be 16 games played and 4 byes. The number of games played is always (n−1) as each game eliminates 1 team. Byes should be scheduled so that no group of teams can possibly get two byes.